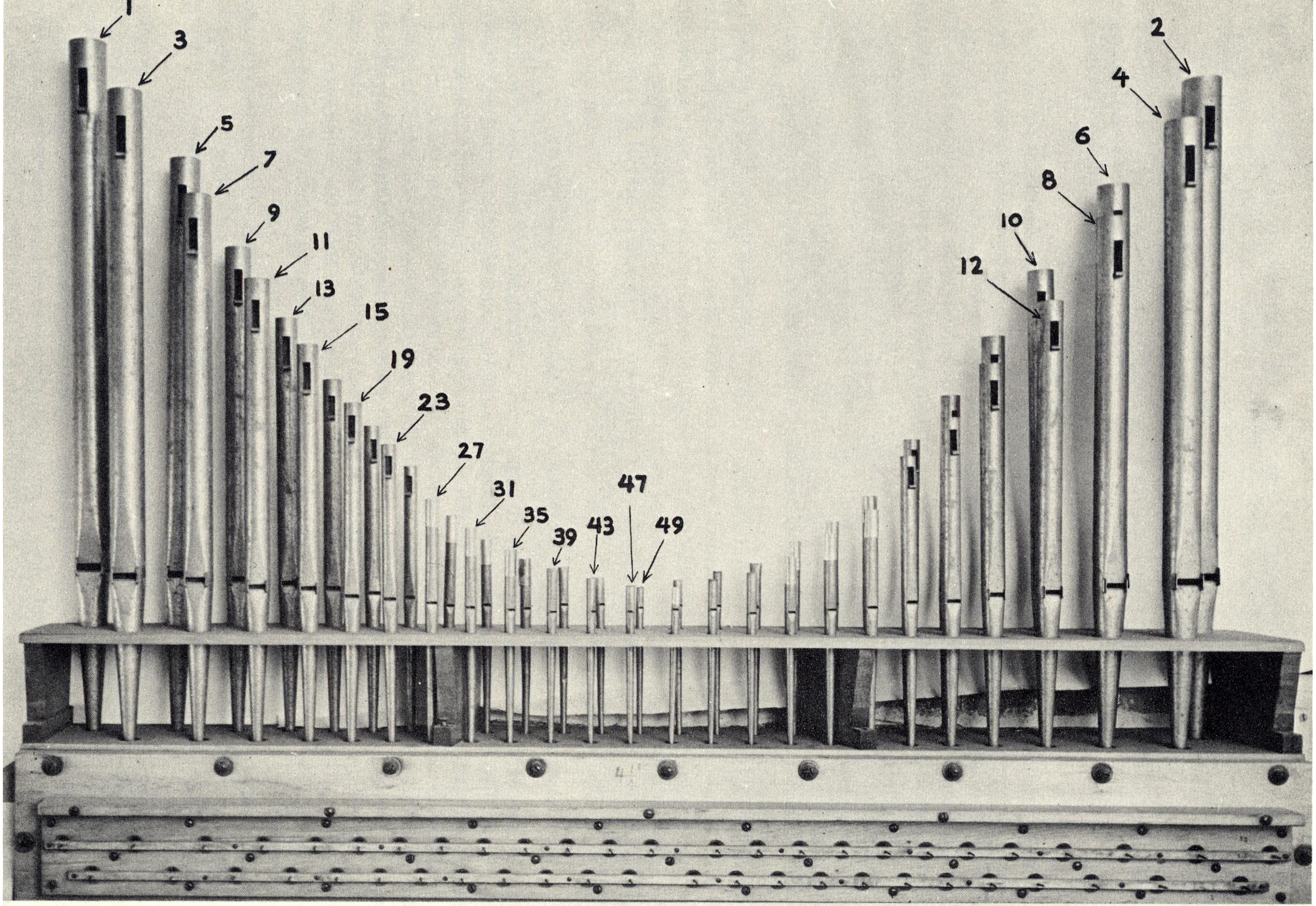


A FOUR OCTAVE ARRAY OF OPEN ORGAN PIPES
256 (NO. 1) TO 4096 (NO. 49) VIBRATIONS PER SECOND

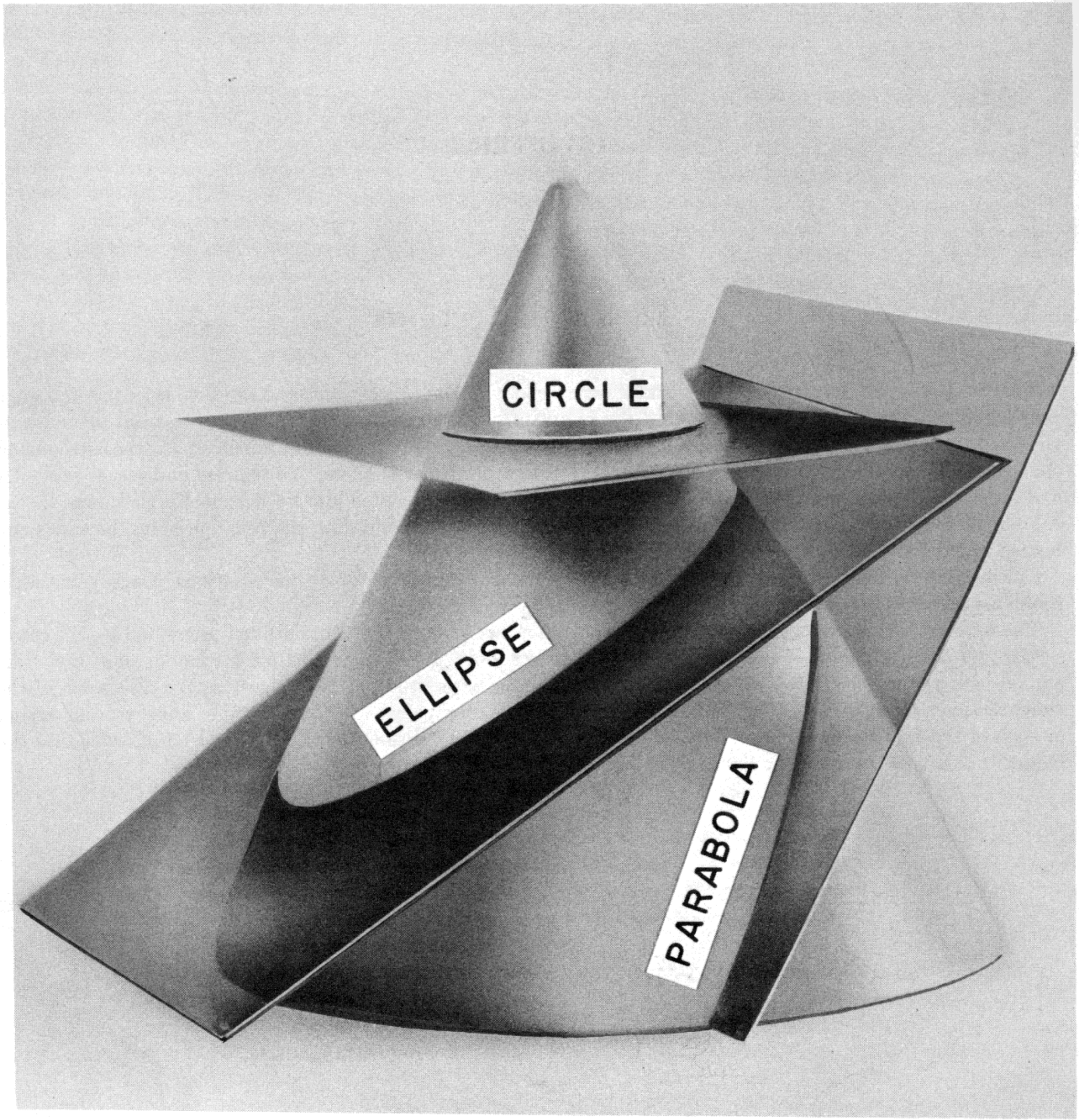


Figure 1. Circle, ellipse and parabola are produced by three intersecting planes. Plane parallel to base provides circle; plane parallel to slant surface provides parabola; plane at angle between the other two planes provides ellipse.

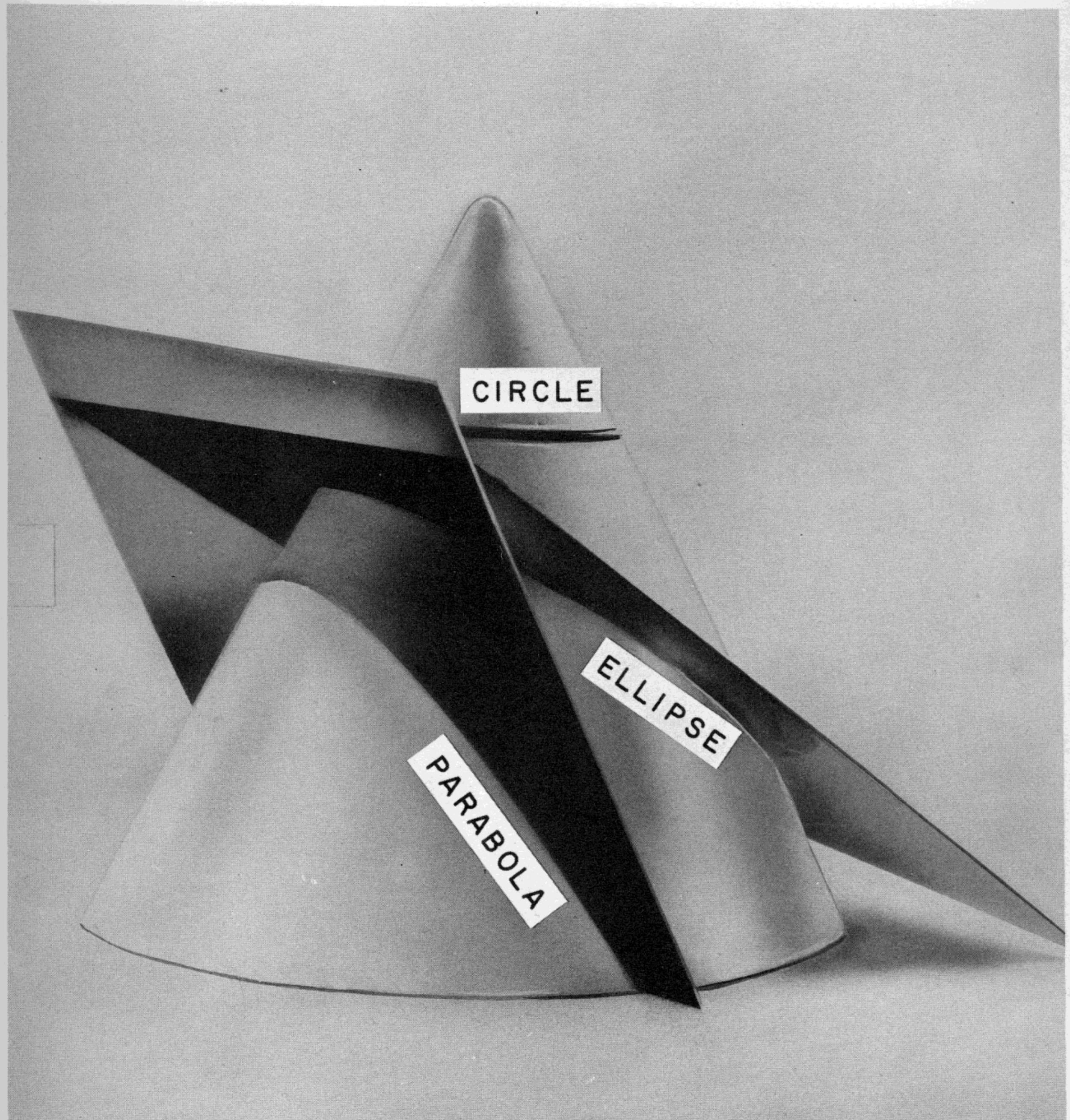


Figure 2. Same model as No. 1, but from different camera angle.

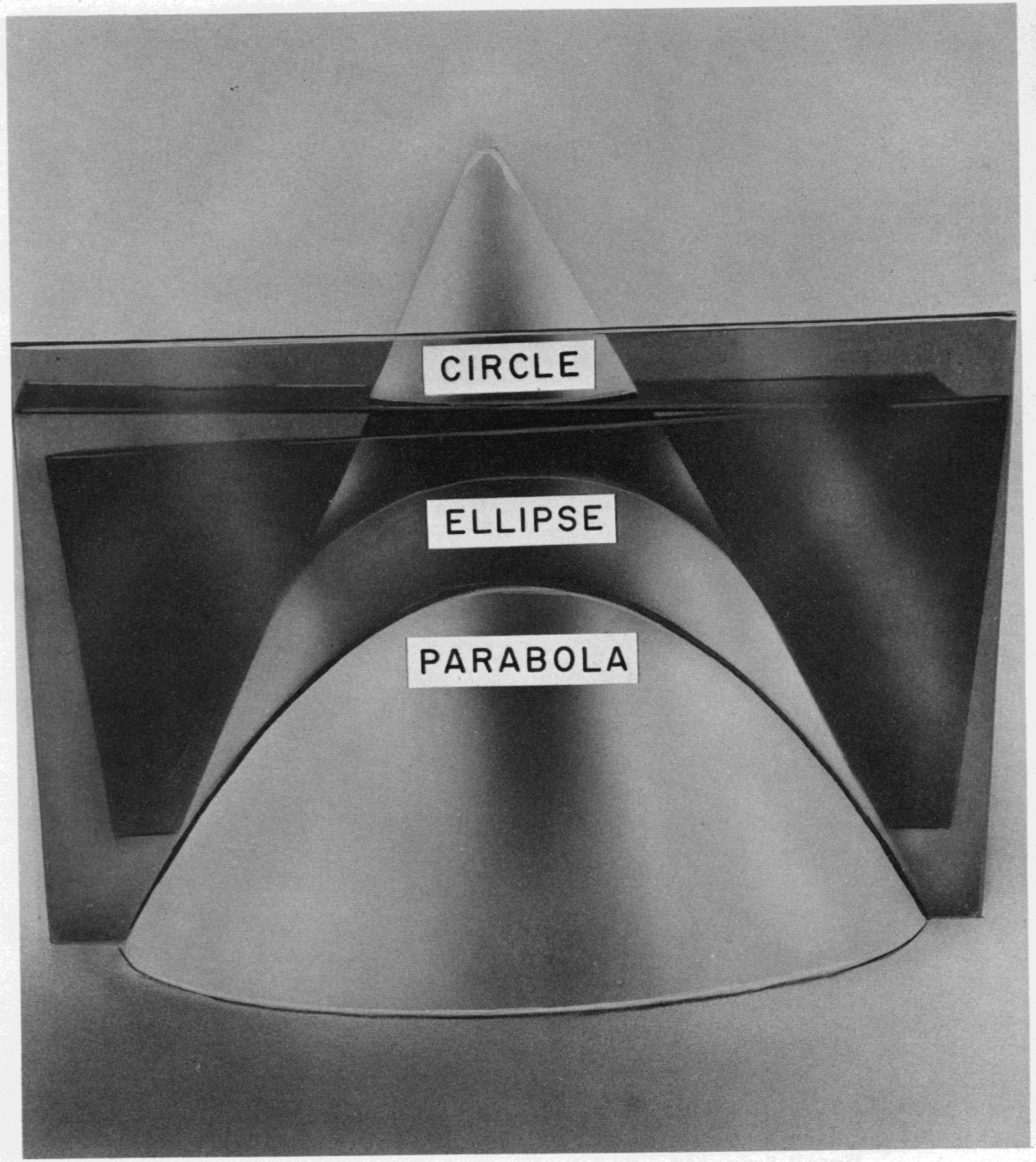


Figure 3. Same model as in No. 1 and No. 2, but from different camera angle.

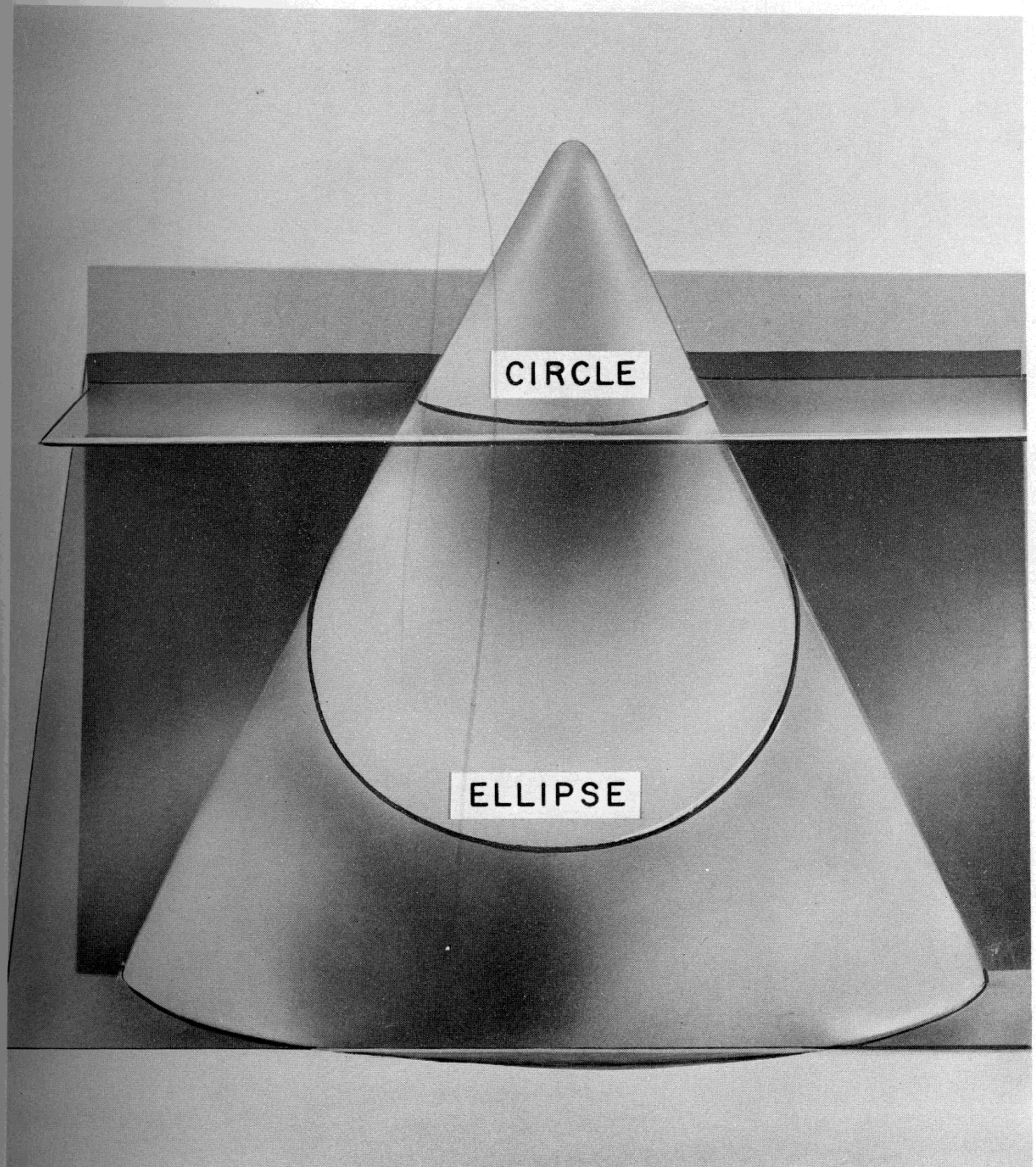


Figure 4. Same model as No. 1, No. 2 and No. 3, but from different camera angle. Parts of circle and ellipse are quite evident.

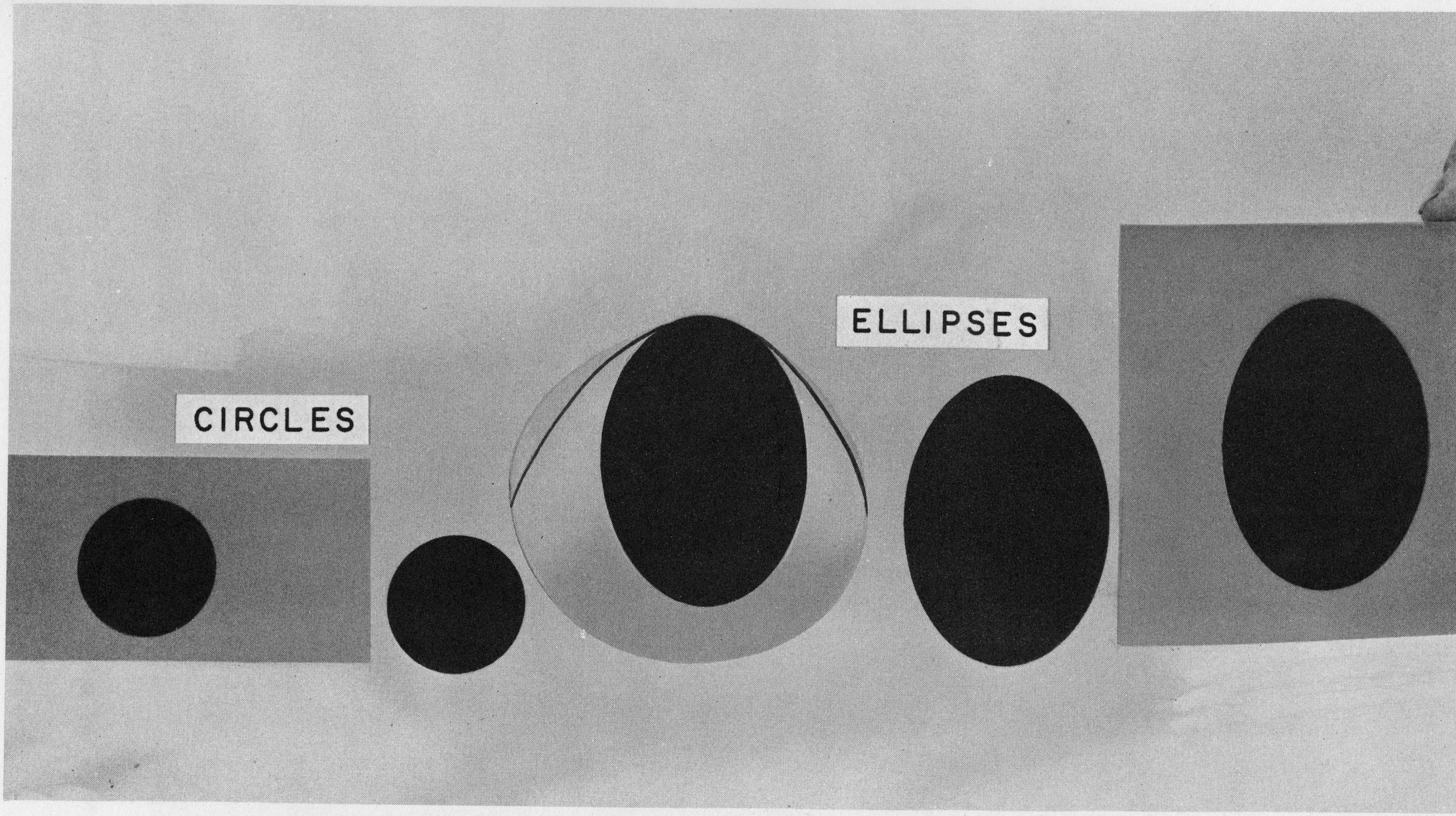


Figure 5. Model No. 1 in dissection. At far left is intersecting plane itself showing circular area. Then next to it on the right is base of circular top section of model. In the center is the elliptical section of model. At the right are the elliptical area on intersecting plane and elliptical area of the model piece taken off from the model at center.

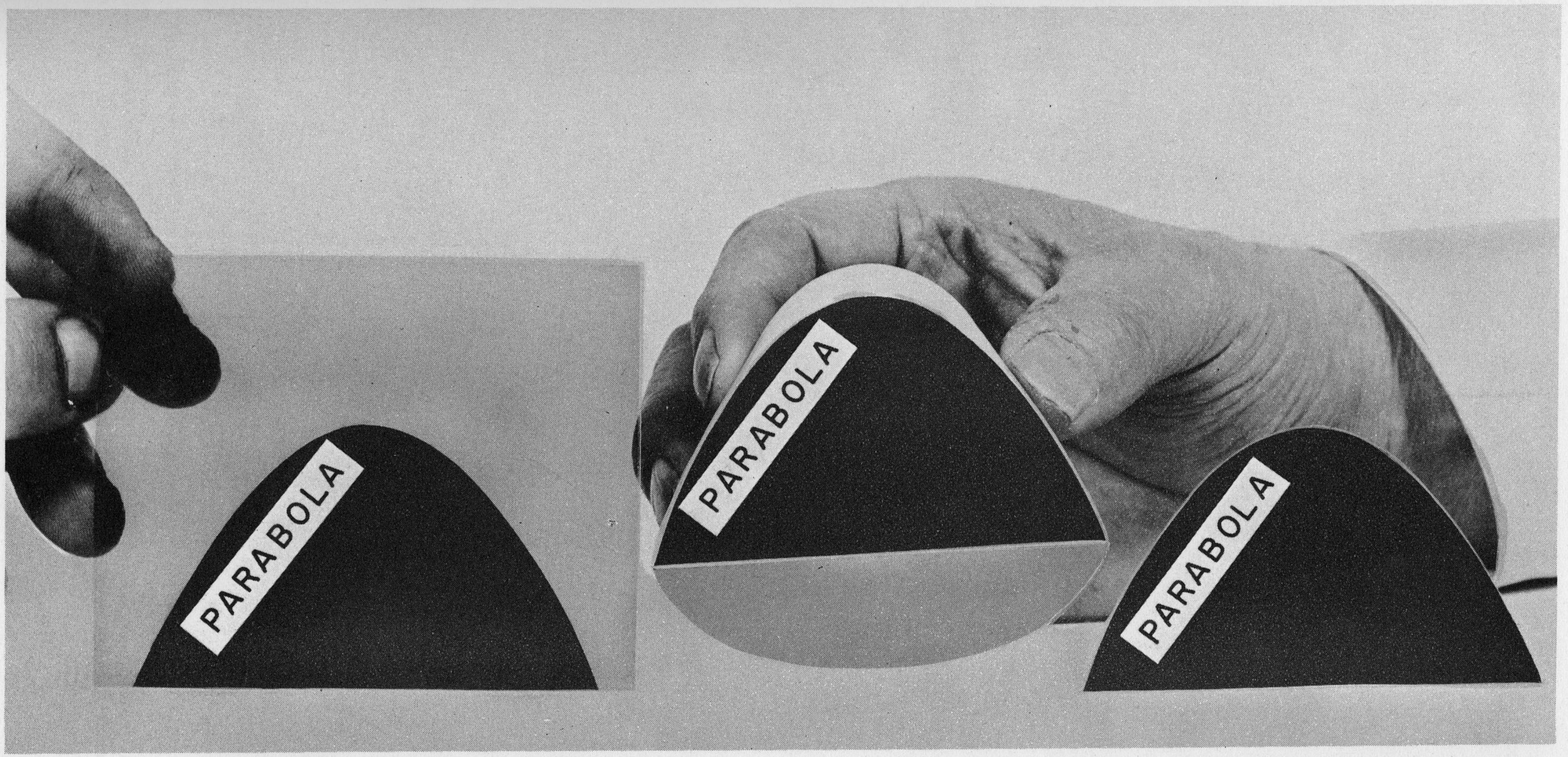


Figure 6. The dissected model of No. 1, showing parabola on intersecting plane, on the main portion of the model and removed model piece.

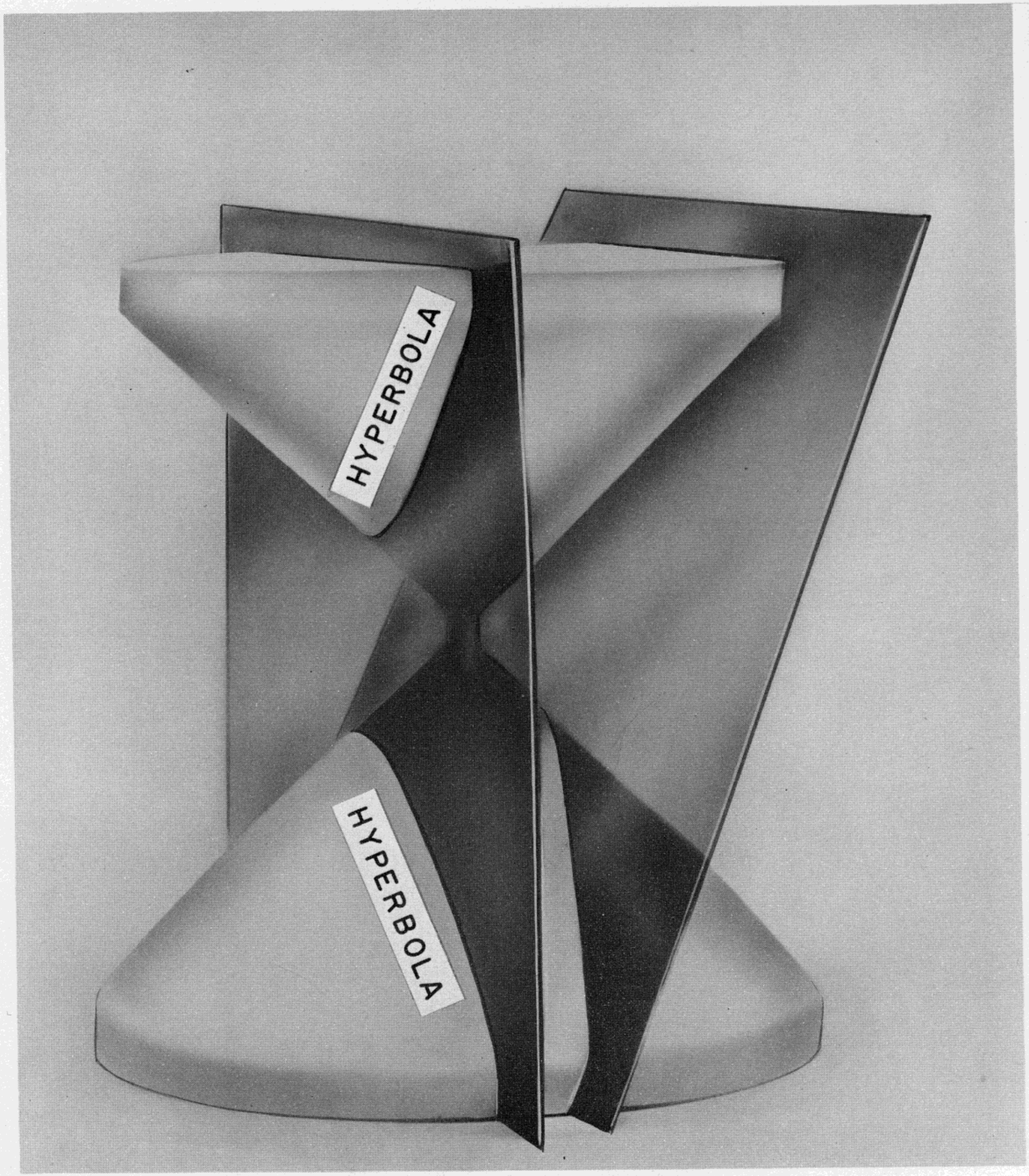


Figure 7. Model and intersecting planes giving hyperbolas in two nappes. The common apex angle is 90°. The base angles of both nappes are 45°. The left intersecting plane is parallel to common vertical axis of the two nappes. The right intersecting plane is not parallel to common vertical axis. The hyperbolas on the right side are not readily evident.

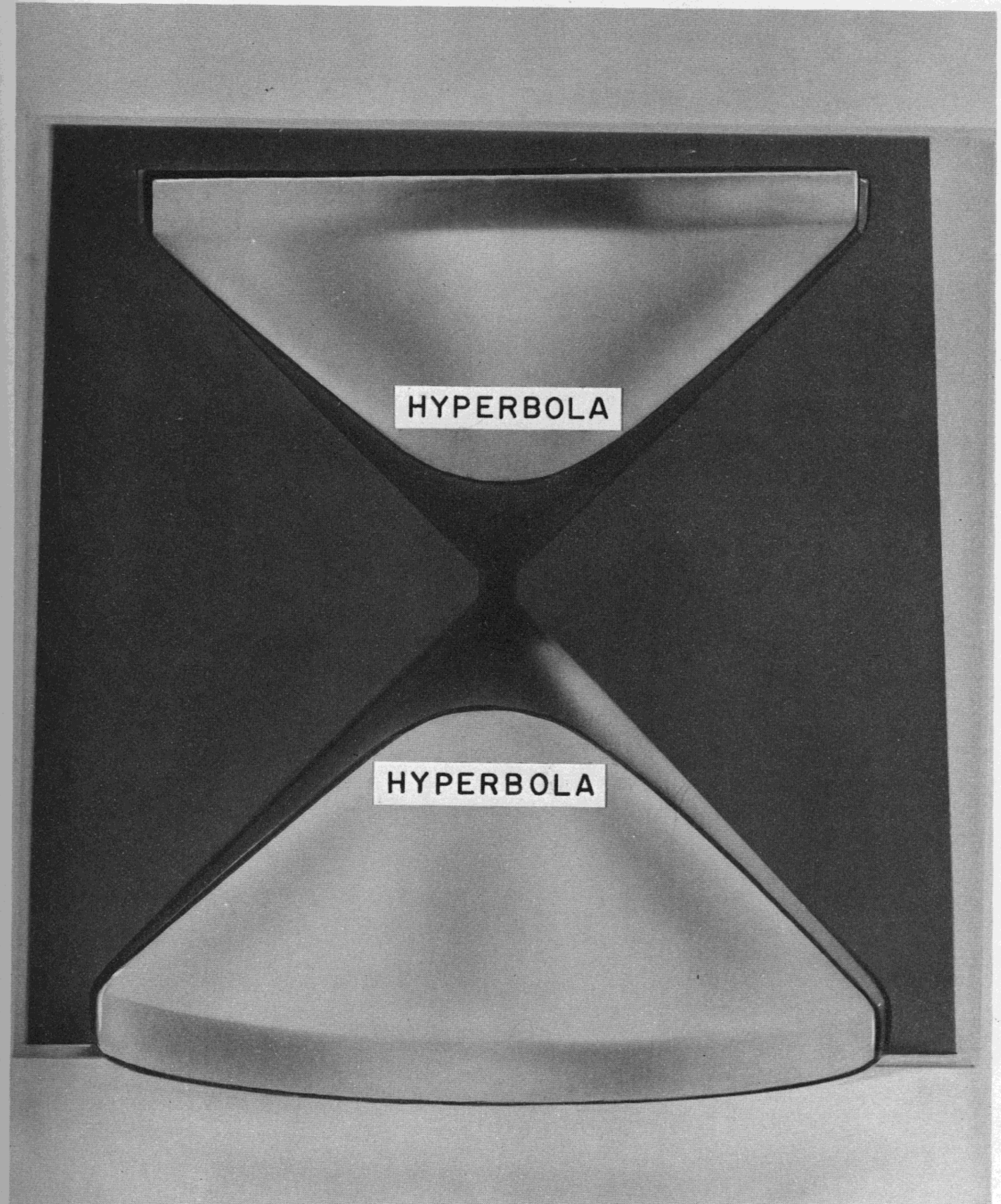


Figure 8. Equilateral hyperbolas obtained from No. 7 by parallel intersecting plane. Eccentricity of both equilateral hyperbolas equals 1.41.

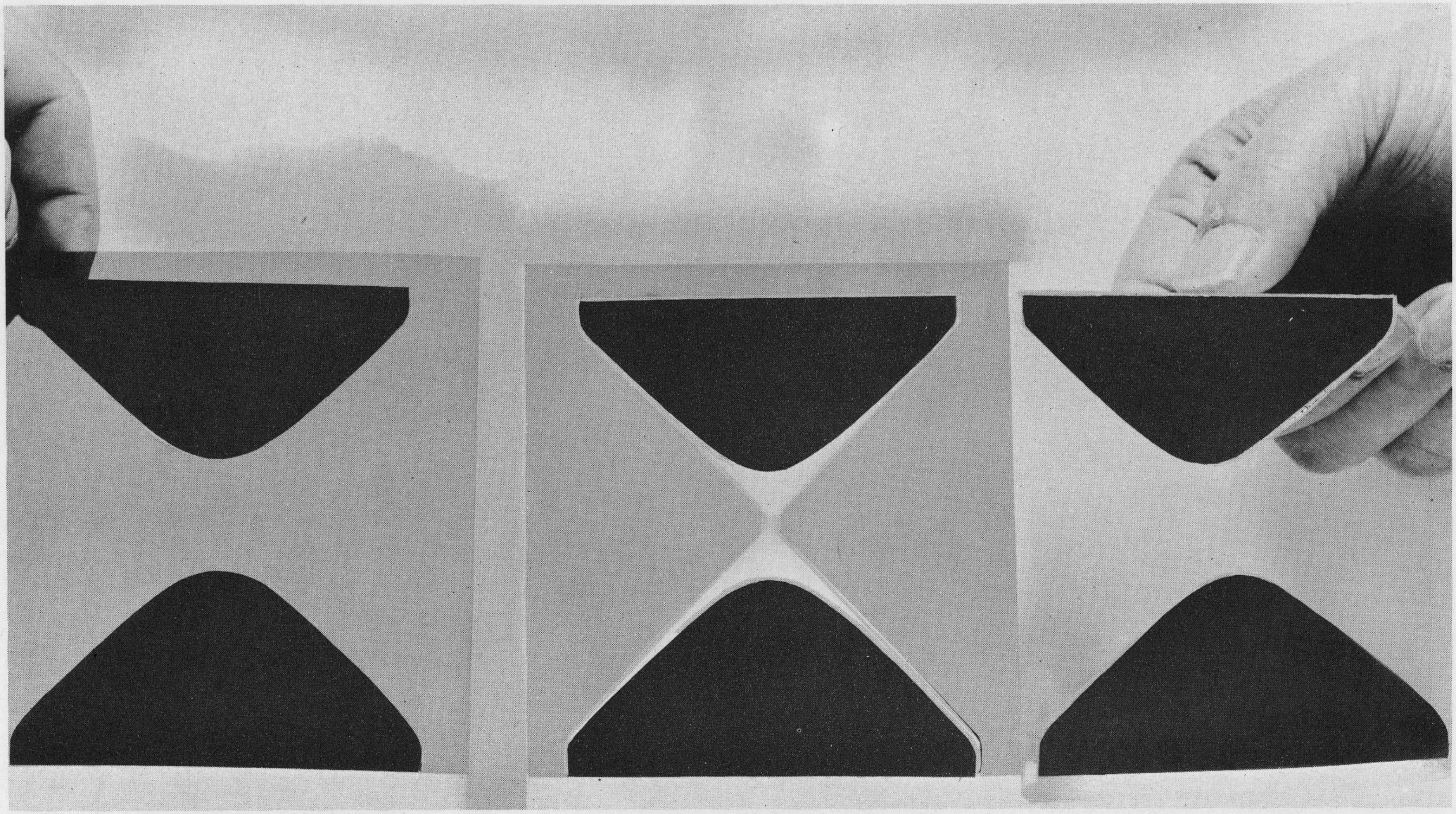


Figure 9. Equilateral hyperbolas obtained by dissection model as shown in No. 8. At left are curves on intersecting planes; in center are curves of main section of model; at right are the curves of removed pieces of model.

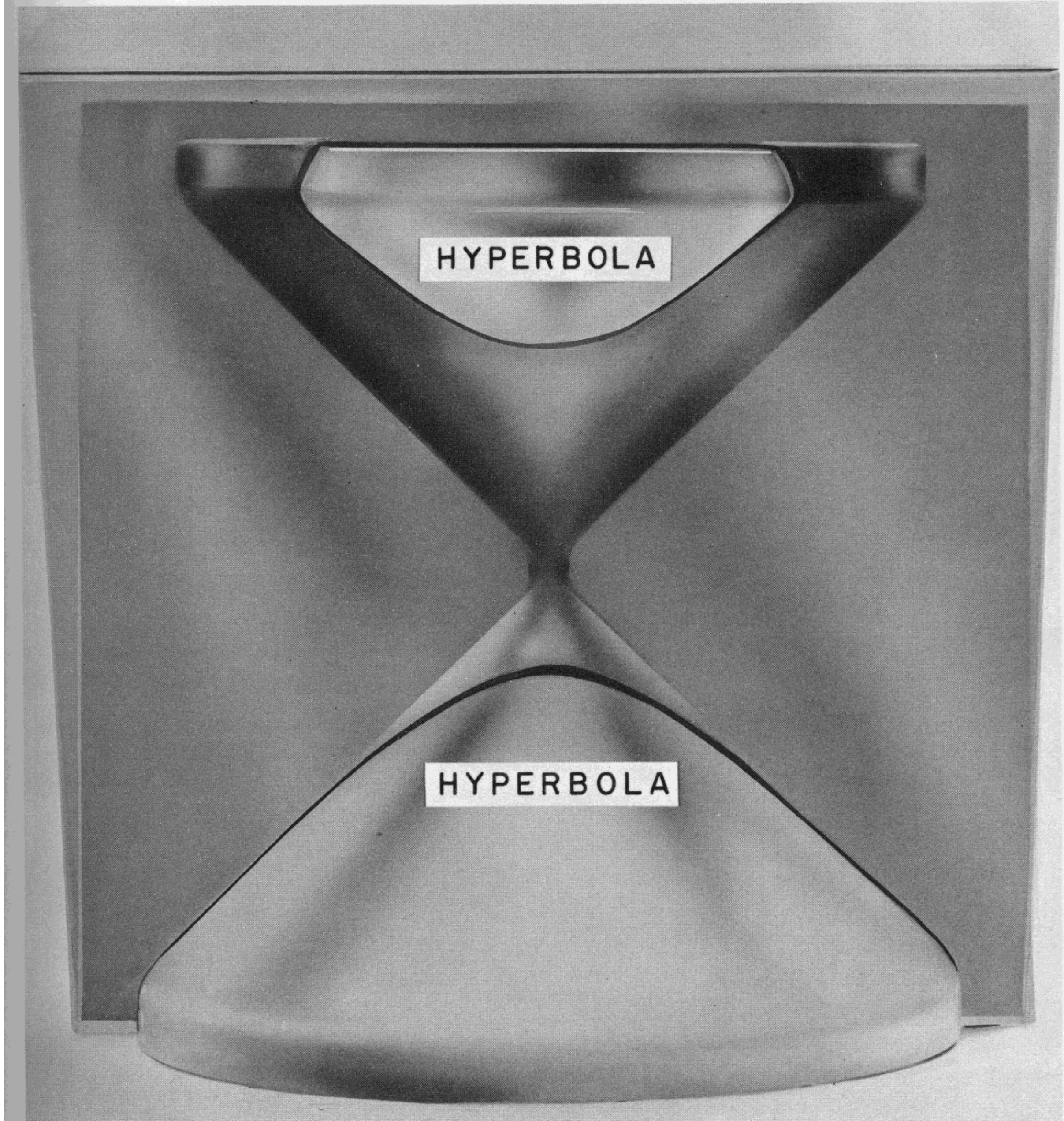


Figure 10. Hyperbolas obtained from No. 7 when intersecting plane is other than parallel to common vertical axis. Their eccentricity is equal, and less than 1.41 and greater than 1. The curves are the same, but are outlined to different lengths along their path.

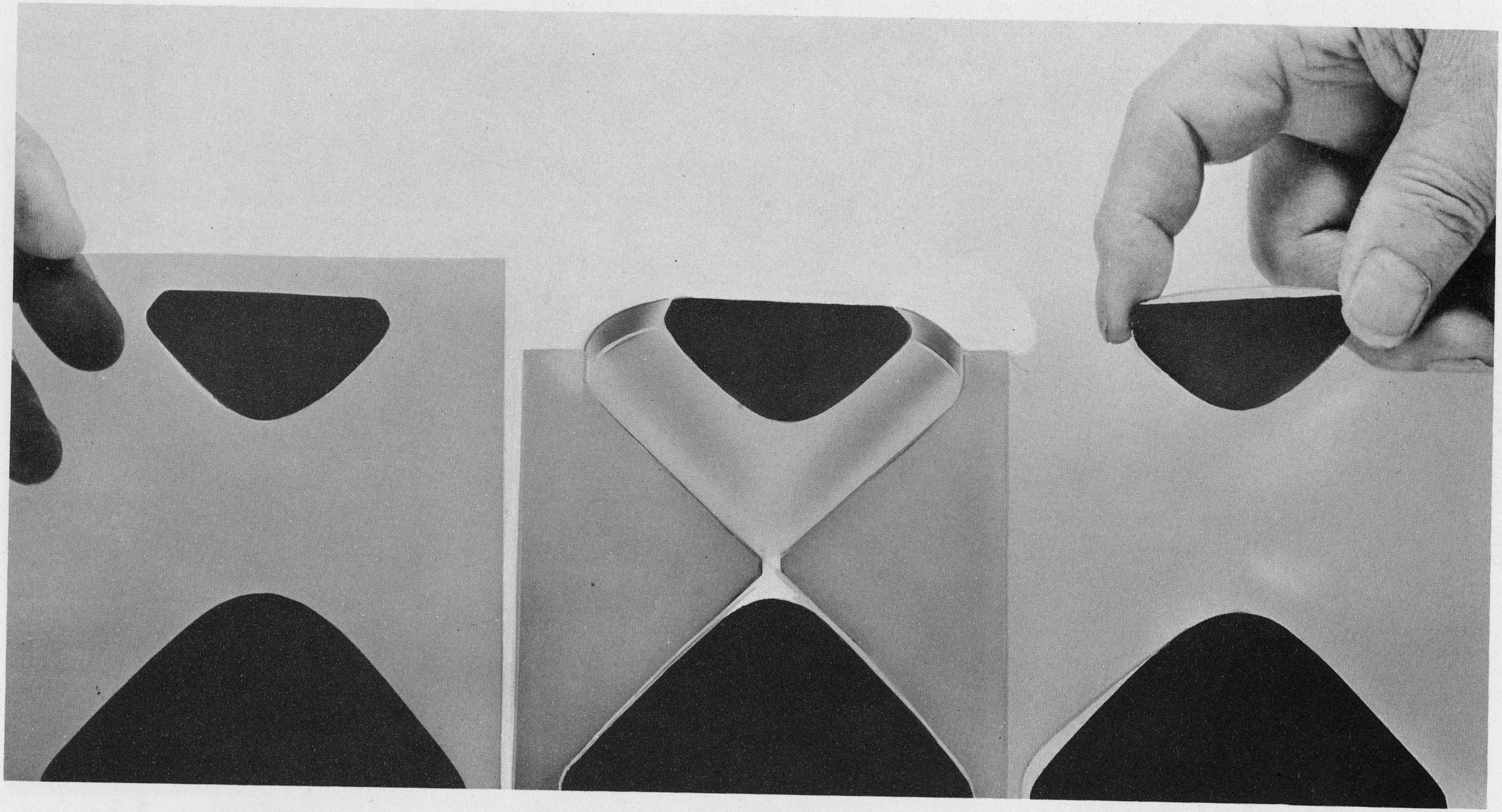


Figure 11. Hyperbolas of different length but same eccentricity as obtained from picture No. 10. At left are the curves as outlined on non-parallel intersecting plane; at center are the curves of main portion of model; at right are the curves as provided by the removed sections of the model.

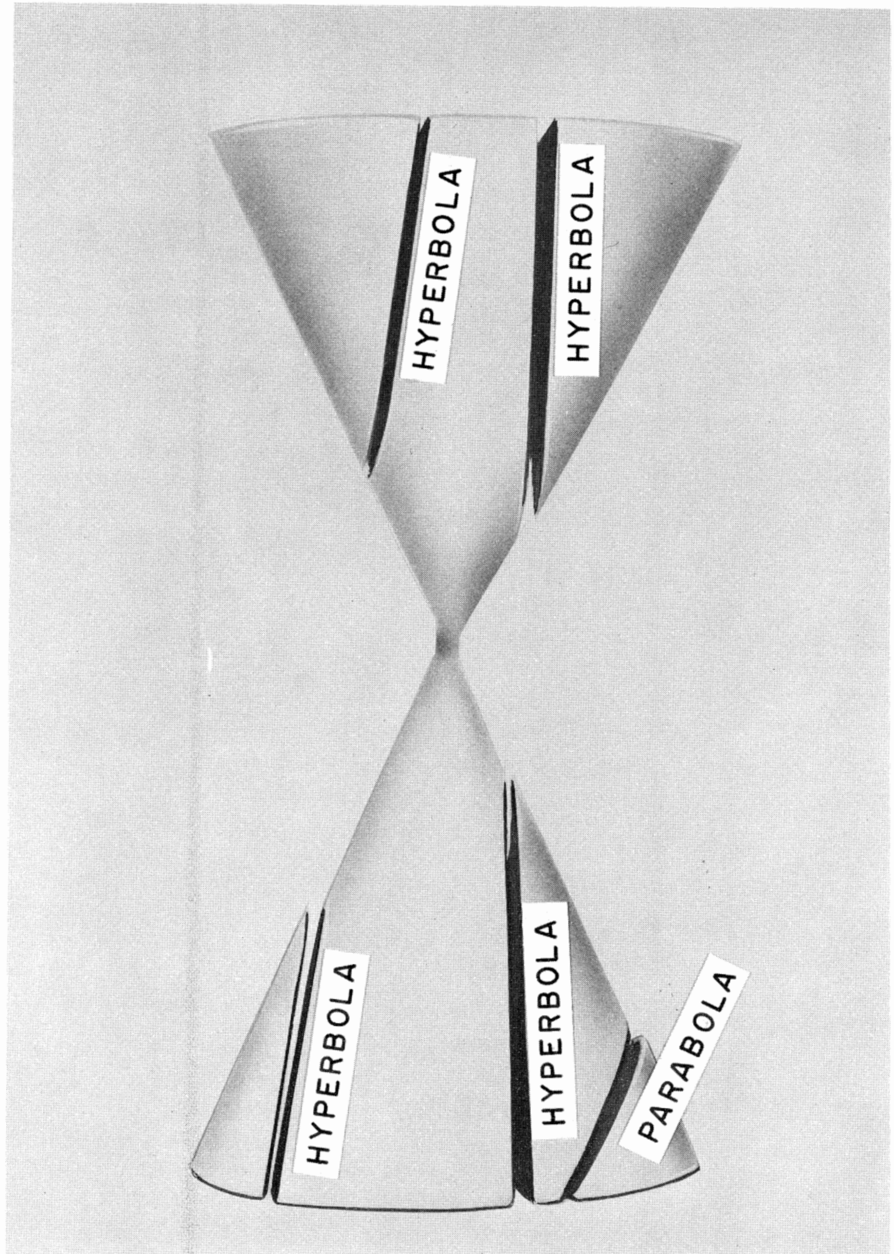


Figure 12. Common apex angle is less than 90 degrees. A small parabola is at lower right-hand corner. Common intersection plane at right is parallel to common vertical axis. The hyperbolas thus created have equal eccentricity and less than 1.41. Intersecting plane at left is aslant with common vertical axis providing two hyperbolas with equal eccentricity which is less than that of other hyperbolas.

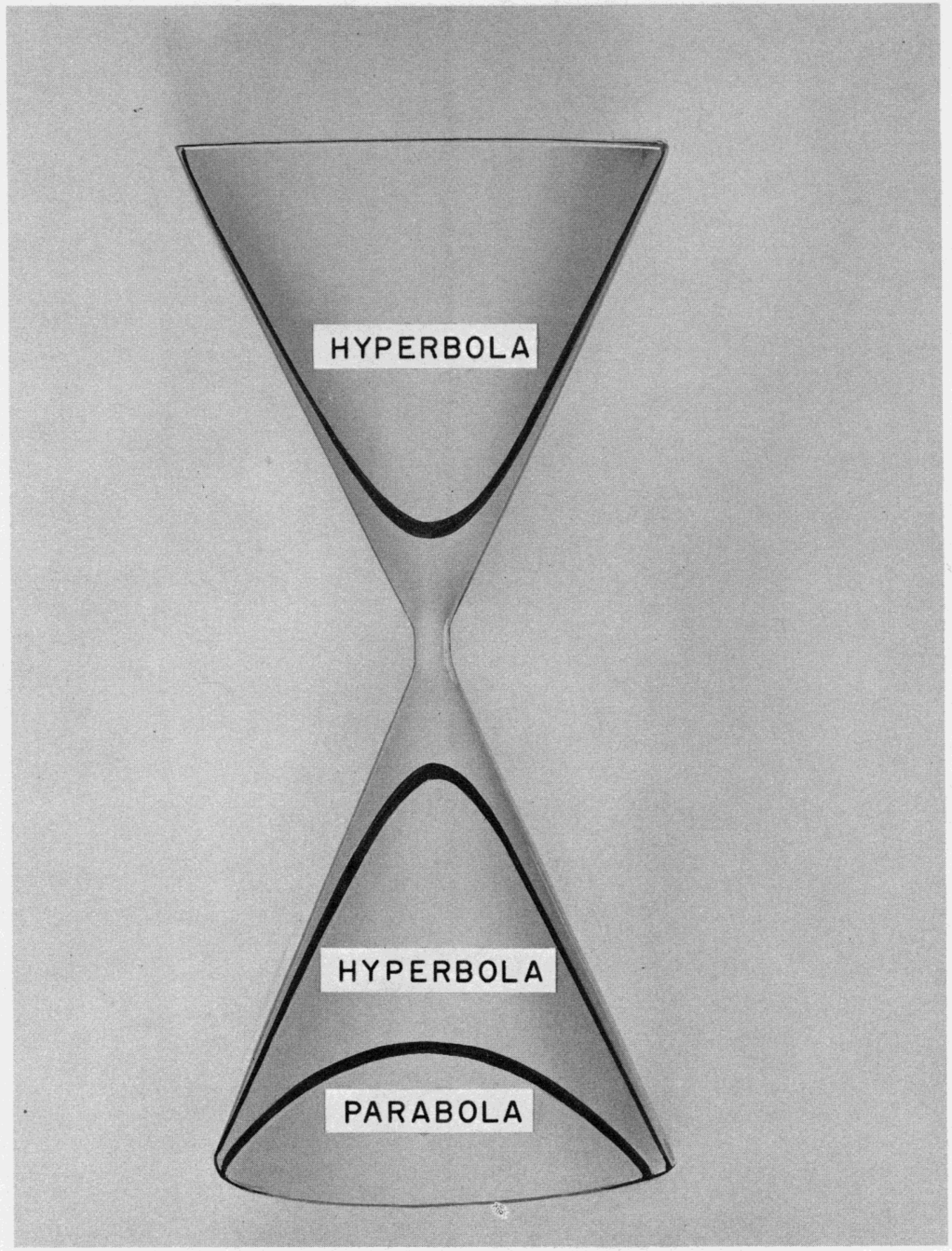
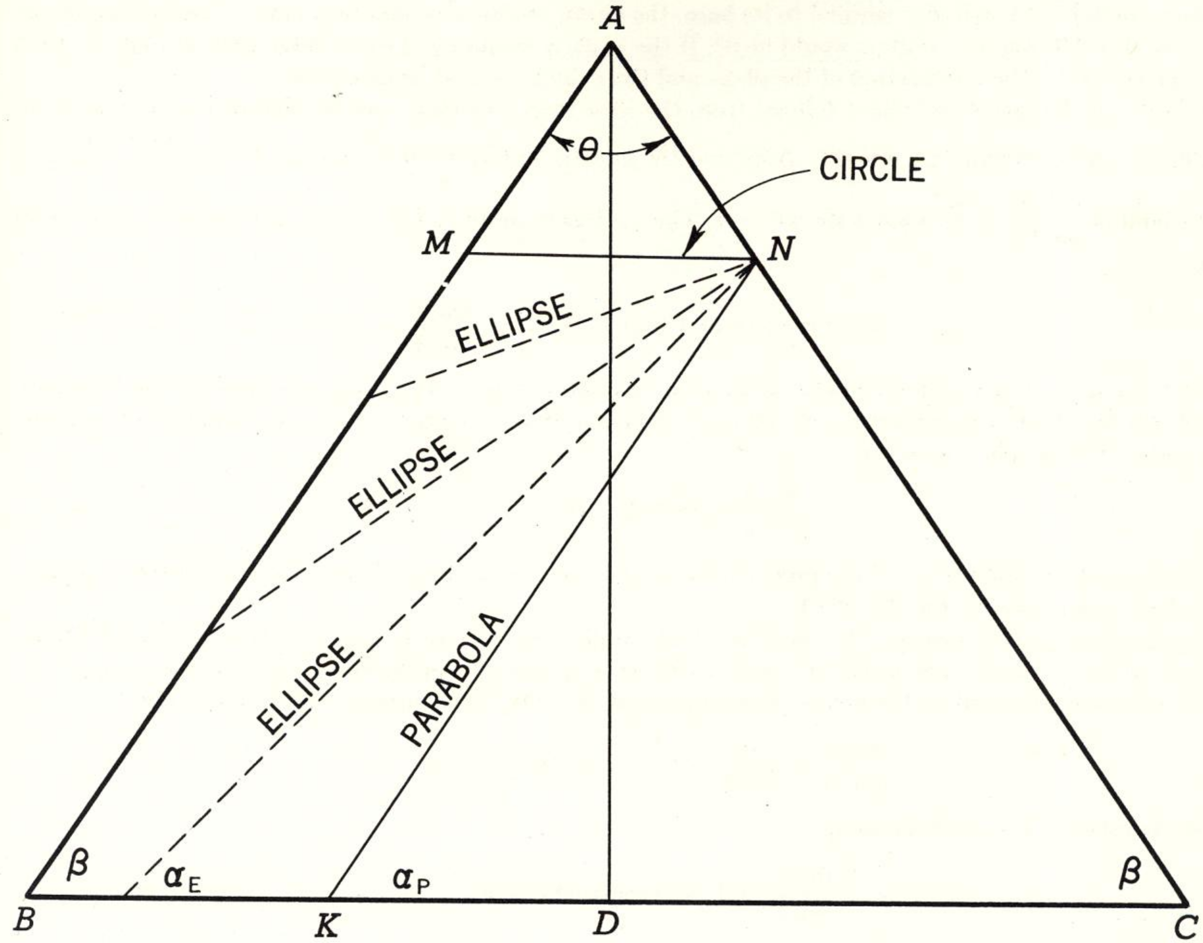


Figure 13. Same as No. 12 from different angle. At the bottom is parabola. The two hyperbolas shown (created by a plane cutting parallel to principal axis) have equal eccentricity. Since common apex angle is small, the two hyperbolas are almost parabolas.

A DRINKING FOUNTAIN PARABOLA



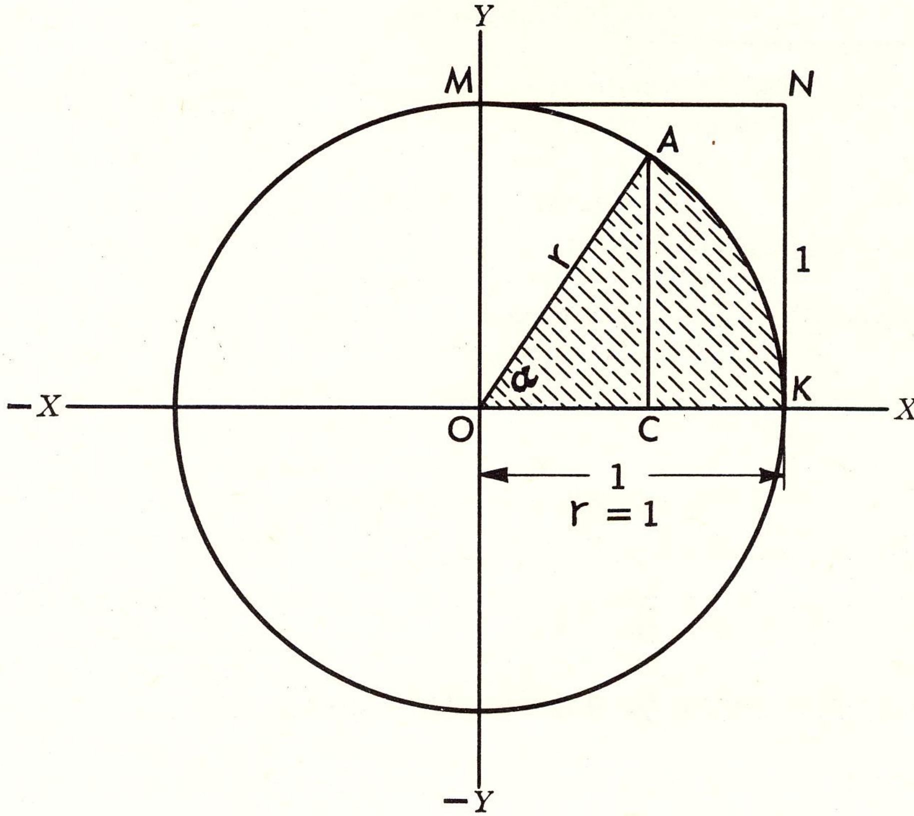
For all Circles, $\alpha = 0 \quad e = 0 = \dfrac{\text{Sin } 0}{\text{Sin } \beta}$

For all Parabolas, $\alpha = \beta \quad e = 1 = \dfrac{\text{Sin } \alpha}{\text{Sin } \beta}$

For all Ellipses, α varies from β to 0

Eccentricities vary as $\dfrac{\text{Sin } \alpha}{\text{Sin } \beta}$ vary = Less than 1

CIRCLE
CIRCULAR RADIAN AND AREA



$$\tan\alpha = \frac{AC}{OC} = \frac{\sin\alpha}{\cos\alpha} = 1.5574$$

$$\frac{\overline{AK}^2}{2} = \text{Circular Sectorial Shaded Area} = \frac{r^2}{2} = \frac{1}{2} = OAKO = \frac{1}{2}\ \text{Square Area}\ (OMNKO = 1)$$

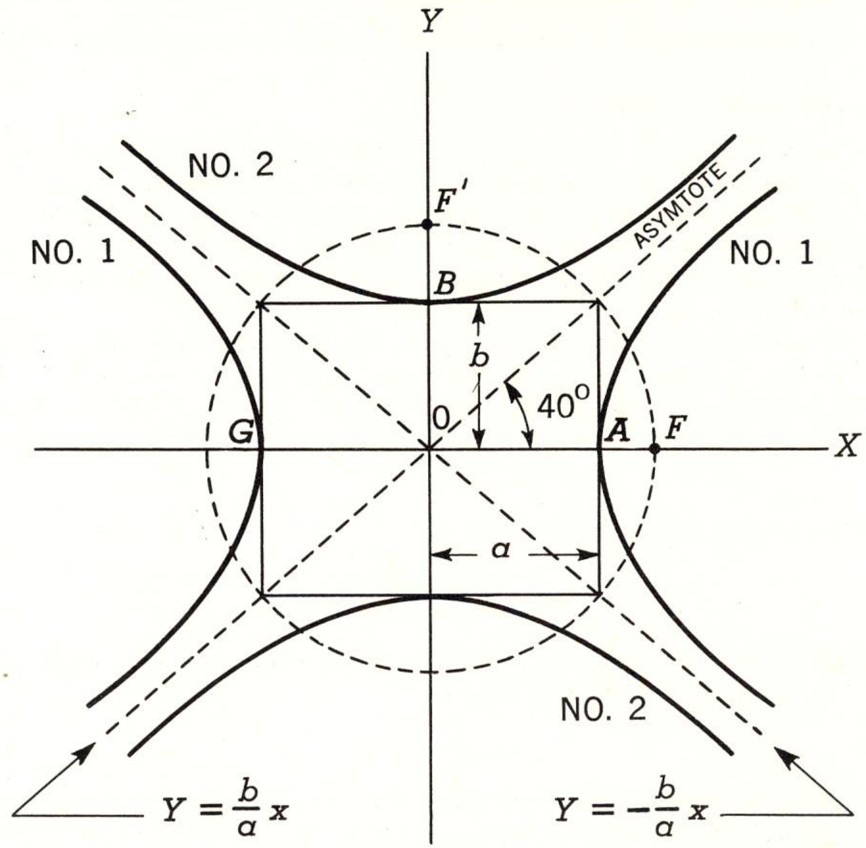
$$\alpha = 1\ \text{Circular Radian} = 57.3^\circ$$

$$\text{Circular Arc}\ (AK) = r = 1$$

$$AC = \sin\alpha = 0.84147$$

$$OC = \cos\alpha = 0.54030$$

$$\cos^2\alpha + \sin^2\alpha = \underline{0.54030}^2 + \underline{0.84147}^2 = r^2 = 1^2 = 1$$



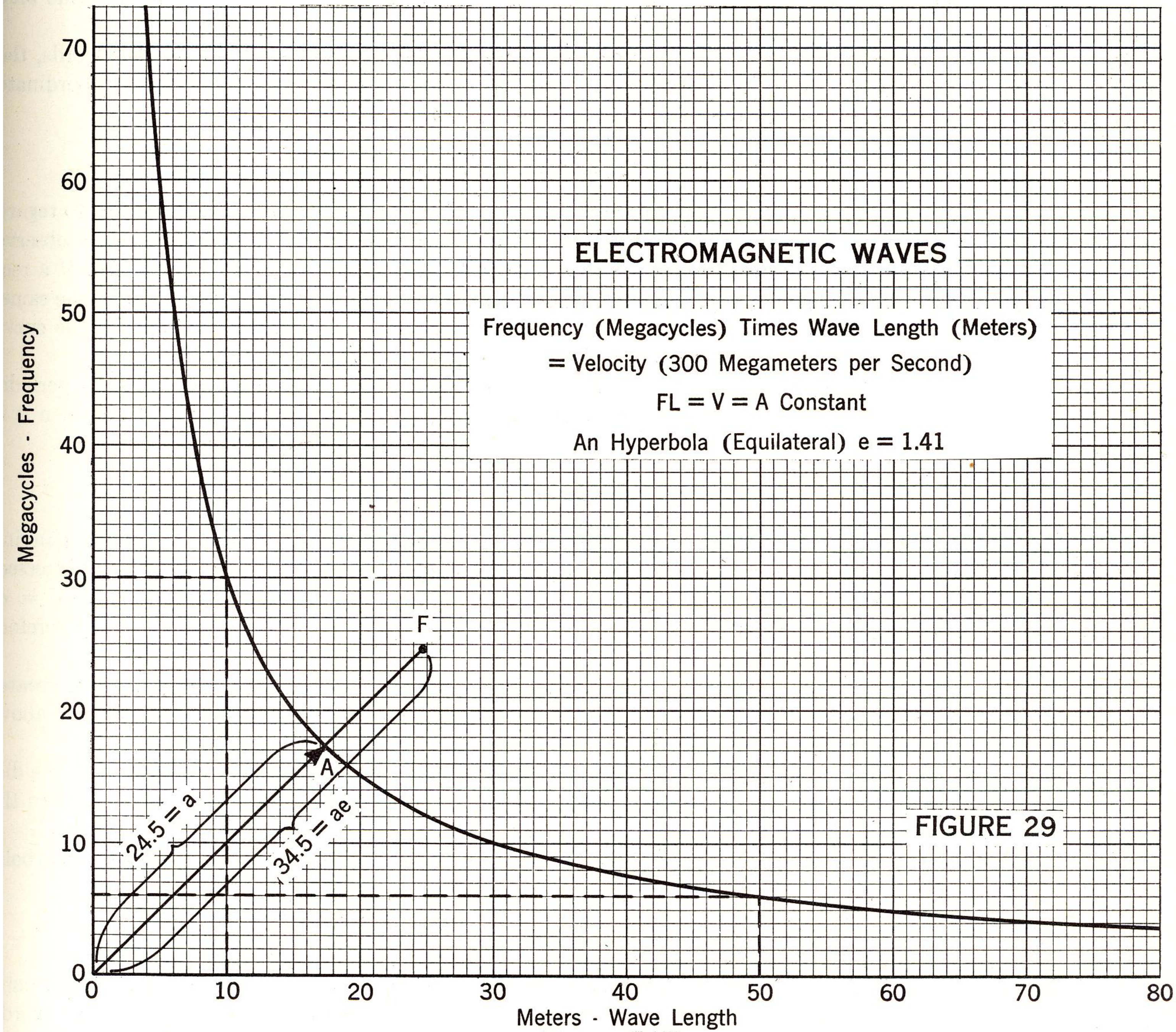
CONJUGATE HYPERBOLAS

"a" Is greater than "b"

$0F = 0F'$

NO. 1 $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ $\qquad e = \dfrac{\sqrt{a^2 + b^2}}{b} = 1.31$

NO. 2 $\dfrac{y^2}{b^2} - \dfrac{x^2}{a^2} = 1$ $\qquad e = \dfrac{\sqrt{a^2 + b^2}}{a} = 1.53$



ELECTROMAGNETIC WAVES
Frequency (Megacycles) Times Wave Length (Meters)
= Velocity (300 Megameters per Second)
FL = V = A Constant
An Hyperbola (Equilateral) e = 1.41
Megacycles - Frequency
Meters - Wave Length
F
A
24.5 = a
34.5 = ae
0
10
20
30
40
50
60
70
80

FIGURE 29

$$+1^{\frac{1}{8}} = \begin{bmatrix} +1^{\frac{1}{4}} = \sqrt[4]{+1} \\ \\ -1^{\frac{1}{4}} = \sqrt[4]{-1} \end{bmatrix} = \sqrt[8]{+1}$$

THE EIGHT SYMBOLIC ELEMENTS

$$+1^{\frac{1}{8}} = \begin{bmatrix} +1 & +j \\ -1 & -j \\ +h & +k \\ -h & -k \end{bmatrix} = \sqrt[8]{+1}$$

$$+1^{\frac{1}{8}} = \begin{bmatrix} +1^{\frac{1}{4}} = \sqrt[4]{+1} \\ \\ -1^{\frac{1}{4}} = \sqrt[4]{-1} \end{bmatrix} = \sqrt[8]{+1}$$

THE EIGHT SYMBOLIC ELEMENTS

$$+1^{\frac{1}{8}} = \begin{bmatrix} +1 & +i \\ -1 & -i \\ +h & +k \\ -h & -k \end{bmatrix} = \sqrt[8]{+1}$$

$$+1]^{\frac{1}{4}} = \begin{bmatrix} +1 & +j \\ -1 & -j \end{bmatrix} = \sqrt[4]{+1}$$

$\boxed{\text{THE FOUR REAL ELEMENTS}}$

$$-1^{\frac{1}{4}} = \begin{bmatrix} +h & +k \\ -h & -k \end{bmatrix} = \sqrt[4]{-1}$$

$\boxed{\text{THE FOUR IMAGE ELEMENTS}}$

$$
+1^{\frac{1}{2}} = \left[\begin{array}{c} +1 \, , \; \left| +1 \right| = 1 \\ \\ -1 \, , \; \left| -1 \right| = 1 \end{array} \right] = \sqrt[2]{+1}
$$

THE TWO D. C. ELEMENTS

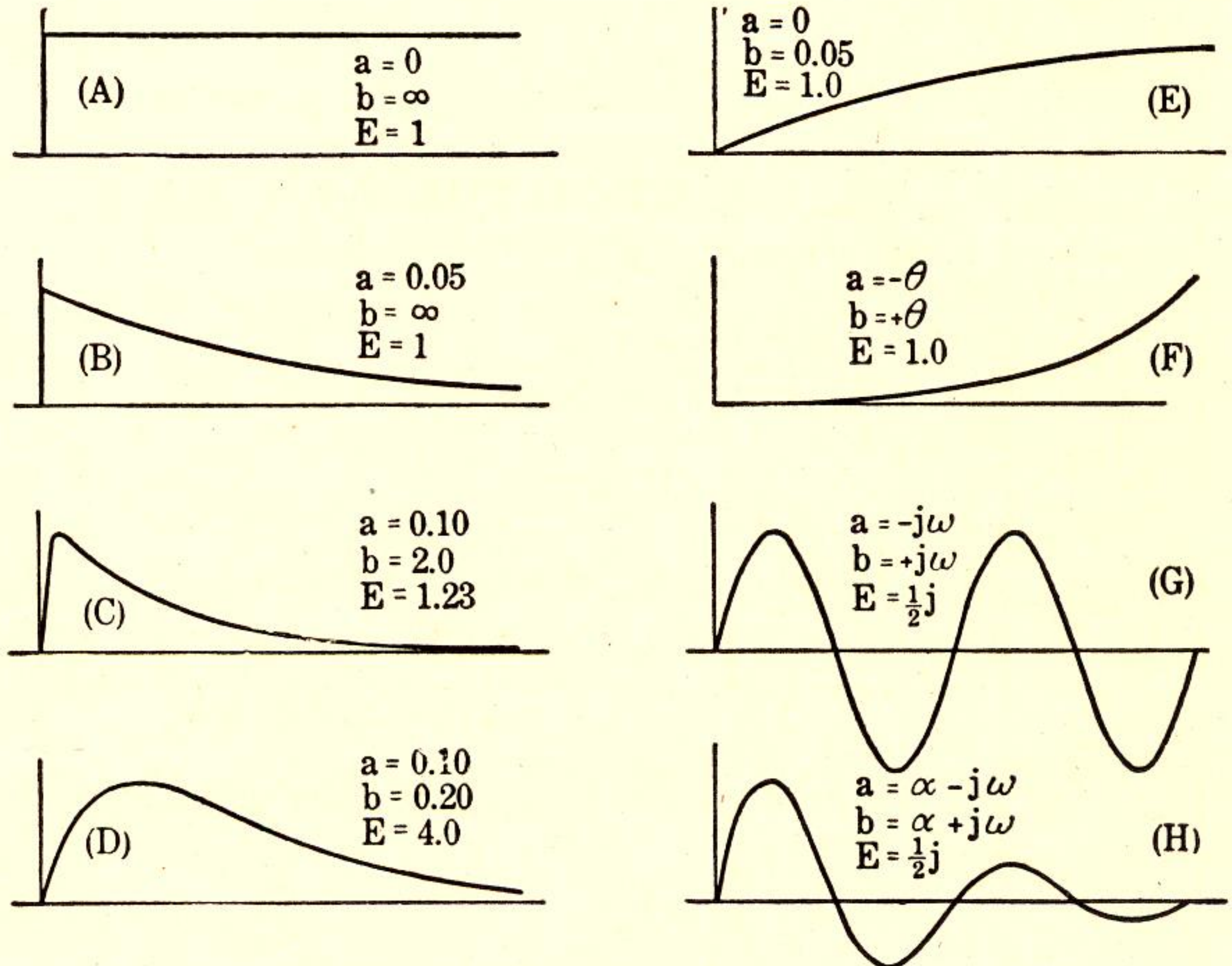


FIG. 4.—Empirical Wave Shapes Given by

$$e = E\,(\varepsilon^{-at} - \varepsilon^{-bt})$$

$$-1^{\frac{1}{2}} = \begin{bmatrix} +j \, , \, \left| +j \right| = 1 \\ -j \, , \, \left| -j \right| = 1 \end{bmatrix} = \sqrt[2]{-1}$$

THE TWO A.C. ELEMENTS

$$+j^{\frac{1}{2}} = \begin{bmatrix} +h, & |+h| = 1 \\ & \\ -h, & |-h| = 1 \end{bmatrix} = \sqrt[2]{+j}$$

THE TWO I.C. ELEMENTS

$$-\dot{\jmath}^{\frac{1}{2}} = \begin{bmatrix} +k, & \left|+k\right| = 1 \\ -k, & \left|-k\right| = 1 \end{bmatrix} = \sqrt[2]{-\dot{\jmath}}$$

$\boxed{\text{THE TWO O.C. ELEMENTS}}$

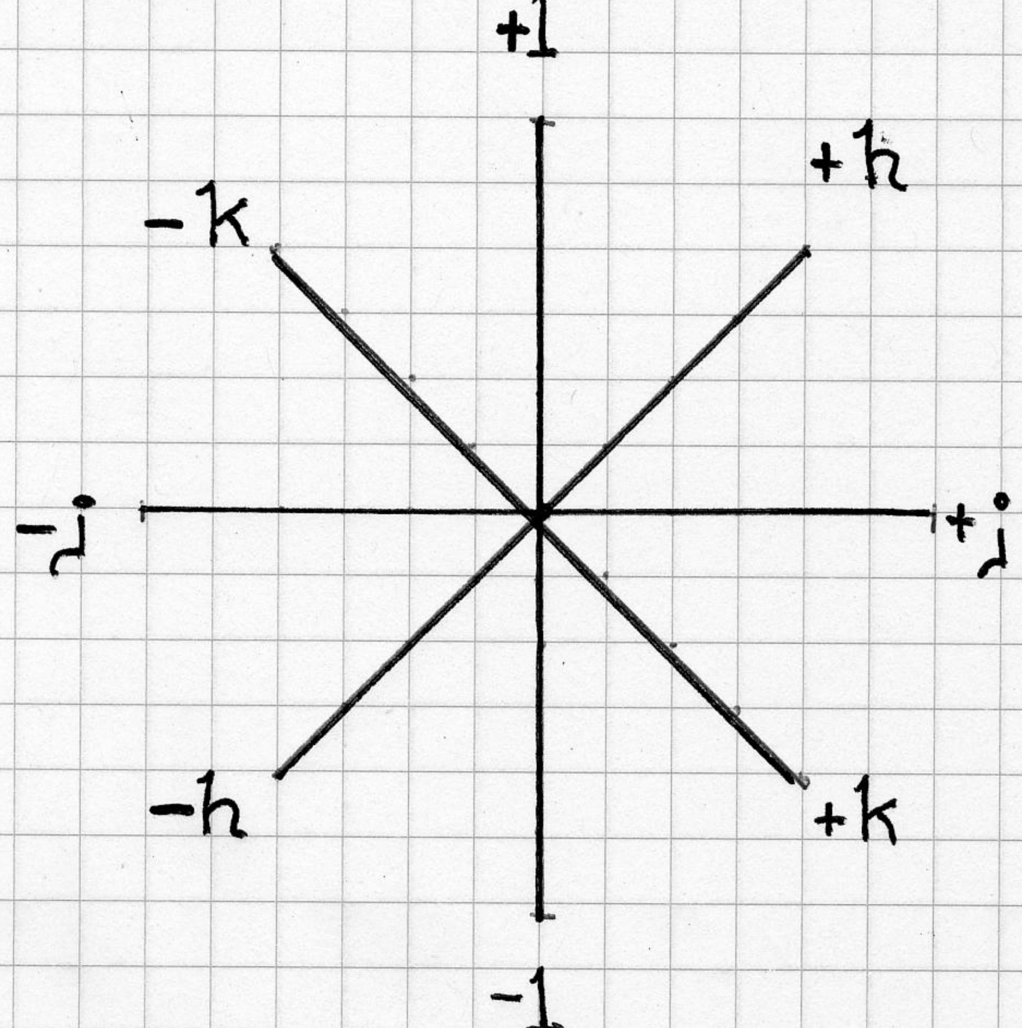
$$1^{1} = \left\| \begin{matrix} +1 & +j \\ -1 & -j \\ +h & +k \\ -h & -k \end{matrix} \right\| = 1$$

THE UNDIVIDED UNITY

+l +h +k +j

−l −h −k −j

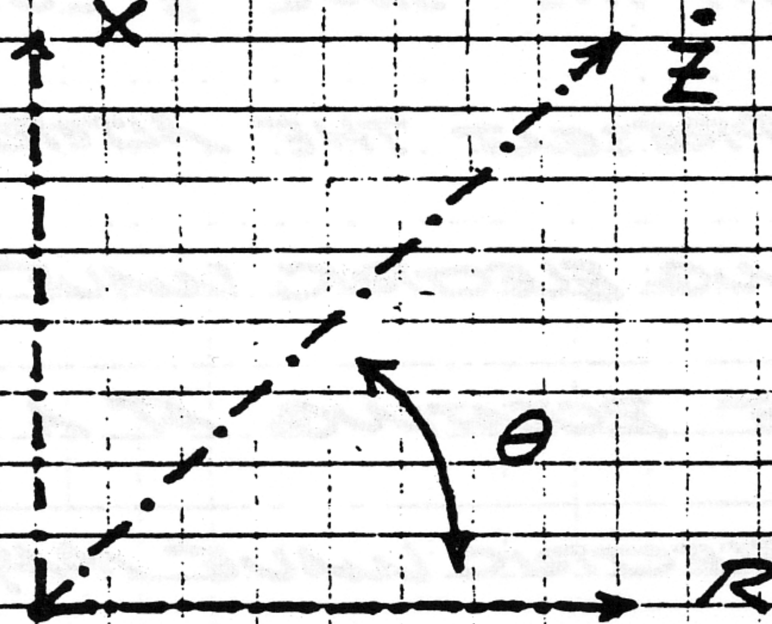
TWO GROUPS OF FOUR



THE EIGHT SYBOLIC POLES

REPRESENTATIONS OF TWO DIMENSIONAL SPACE, THAT IS, A PLANE SURFACE. THE CONCEPT OF A "SURFACE OF TIME" IS OF LITTLE VALUE FOR THE THEORETICAL INVESTIGATION OF ELECTRIC WAVES SINCE TIME IS AN AXIAL DIMENSION, TYPICALLY GIVEN AS POINTS ON A LINE.

CONSIDER THE ADDITION OF ELECTRIC RESISTANCE, R, IN OHMS, AND MAGNETIC INDUCTIVE REACTANCE, X, IN HENRYS PER SECOND, (Hz) L. THE USUAL REPRESENTATION IS GIVEN BY



$$|\dot{Z}| = (R^2 + X^2)^{1/2}$$

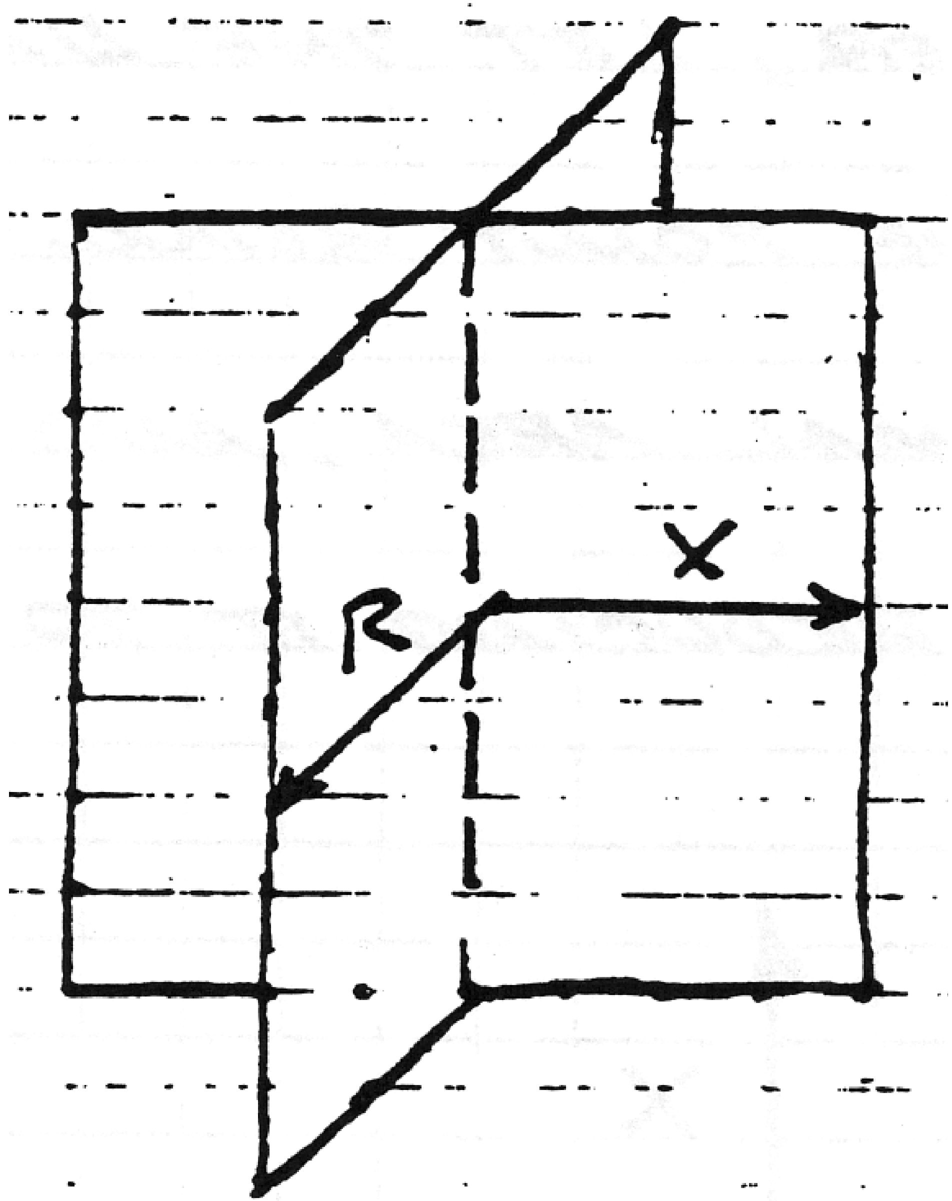
$$\theta = \tan^{-1}\frac{X}{R}$$

THE RESISTANCE OF AN ELECTRIC SYSTEM IS, HOWEVER A PROPERTY OF THE SYSTEM THAT IS FREQUENCY, OR TIME, INVARIANT. THUS RESISTANCE IS A SCALAR QUANITY INDEPENDENT OF THE TIME RATE OF VARIATION OF THE APPLIED ELECTRIC WAVE. RESISTANCE THEN IS NOT A VECTOR QUANITY AS PORTRAID IN FIGURE 1.

THE REACTANCE OF AN ELECTRIC SYSTEM IS ITS MAGNETIC INDUCTANCE, L, MULTIPLIED BY THE TIME RATE OF VARIATION

OF THE APPLIED ELECTRIC WAVE, (X_C), AND BY EQUATION (8) IT IS A TIME DEPENDENT QUANITY ASSOCIATED WITH A QUADRATURE VERSOR. THUS REACTANCE IS ALSO NOT A VECTOR QUANITY, HENCE THE ADDITION OF RESISTANCE & REACTANCE IS NOT PROPERLY REPRESENTED BY A VECTOR DIAGRAM SUCH AS FIGURE 1. THE GRAPHICAL METHOD THEN REALLY SERVES AS A FORM OF COMPUTING APPARATUS FOR CALCULATING PURPOSES AND IS INCAPABLE OF PROVIDING THE PROPER REPRESENTATION OF THE ELECTRIC WAVE REQUIRED FOR THEORETICAL INVESTIGATION.

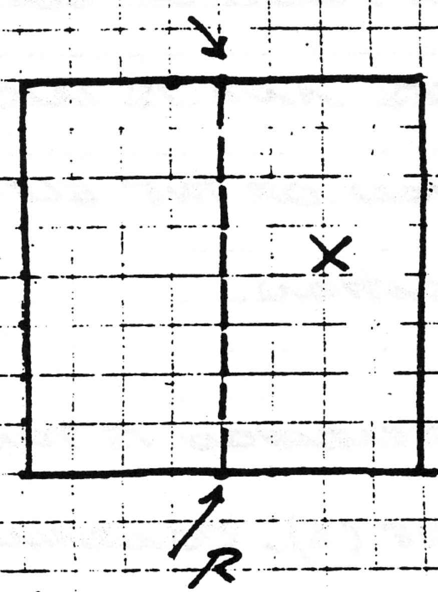
3) AN ANALOGOUS REPRESENTATION IS TWO PERPENDICULAR PLANES IN SPACE, FIGURE (2), CONTAINING THE VECTORS OF FIGURE (1).



X
R

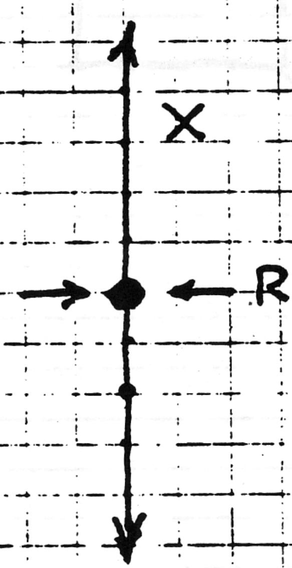
IF THE VIEWER FACES ONE OF THE TWO PLANES STRAIGHT ON, PLANE X FOR EXAMPLE, THEN THE QUADRATURE PLANE, R, HAVING NO THICKNESS BY DEFINITION OF A PLANE SURFACE, DISSAPEARS FROM VIEW,

FIG (3)



GOING ONE STEP FURTHER, LET THE LINE, R, BE REDUCED TO A SINGLE POINT, THE POINT BEING THE THICKNESS OF A PLANE OF INFINITESIMAL AREA, AND LET THE PLANE, X, BE VIEWED EDGEWISE REDUCING IT TO A SINGLE LINE, AS SHOWN IN FIGURE (4).

FIG (4)



2

THE RESULT IS A SINGLE POINT, R, IN THE CENTER OF A LINE, X. HENCE, THE POINT, R, REPRESENTS THE RESISTANCE OF THE ELECTRIC SYSTEM, AND THE AMOUNT OF RESISTANCE IS GIVEN BY THE "WEIGHT" OF THE POINT. THE LINE, X, REPRESENTS THE REACTANCE OF THE ELECTRIC SYSTEM, AND THE AMOUNT OF REACTANCE IS GIVEN BY THE LENGTH OF THIS LINE.

DESPITE ITS SOMEWHAT CONTRIVED NATURE THE REPRESENTATION OF FIGURE (4) ~~IT~~ IS MORE REPRESENTITIVE OF THE ELECTRIC PHENOMENON THAN IS FIGURE (1).

C) SYMBOLIC REPRESENTATION OF ALTERNATING ELECTRIC WAVES.

1) SINCE THE AFOREMENTIONED METHODS ARE ONLY USABLE FOR SITUATIONS INVOLVING FEW QUANTITIES AND ARE MISREPRESENTITIVE OF THE ELECTRIC RELATIONS TO WHICH THEY ARE APPLIED, A METHOD IS THEREFORE DESIRABLE THAT IS CAPABLE OF EXTENSIVE CALCULATION WHILE RETAINING A BASIC SIMPLE FORM REPRESENTITIVE OF THE WAVE.

IT IS WELL KNOWN THAT THE QUADRATURE ANGLE, 90° OR $\pi/2$ RADIANS, REPRESENTS A FUNDAMENTAL RELATION IN A.C. THEORY. SINCE 90° IS ONE FOURTH OF A COMPLETE CYCLE, THE COMPLETE ALTERNATING ELECTRIC WAVE IS REPRESENTED IN ITS ENTIRETY BY FOUR QUADRANTS OF ROTATION.

THESE ROTATIONS ARE REPRESENTED BY FIGURES (5), (6), & (7)

FIG (5)

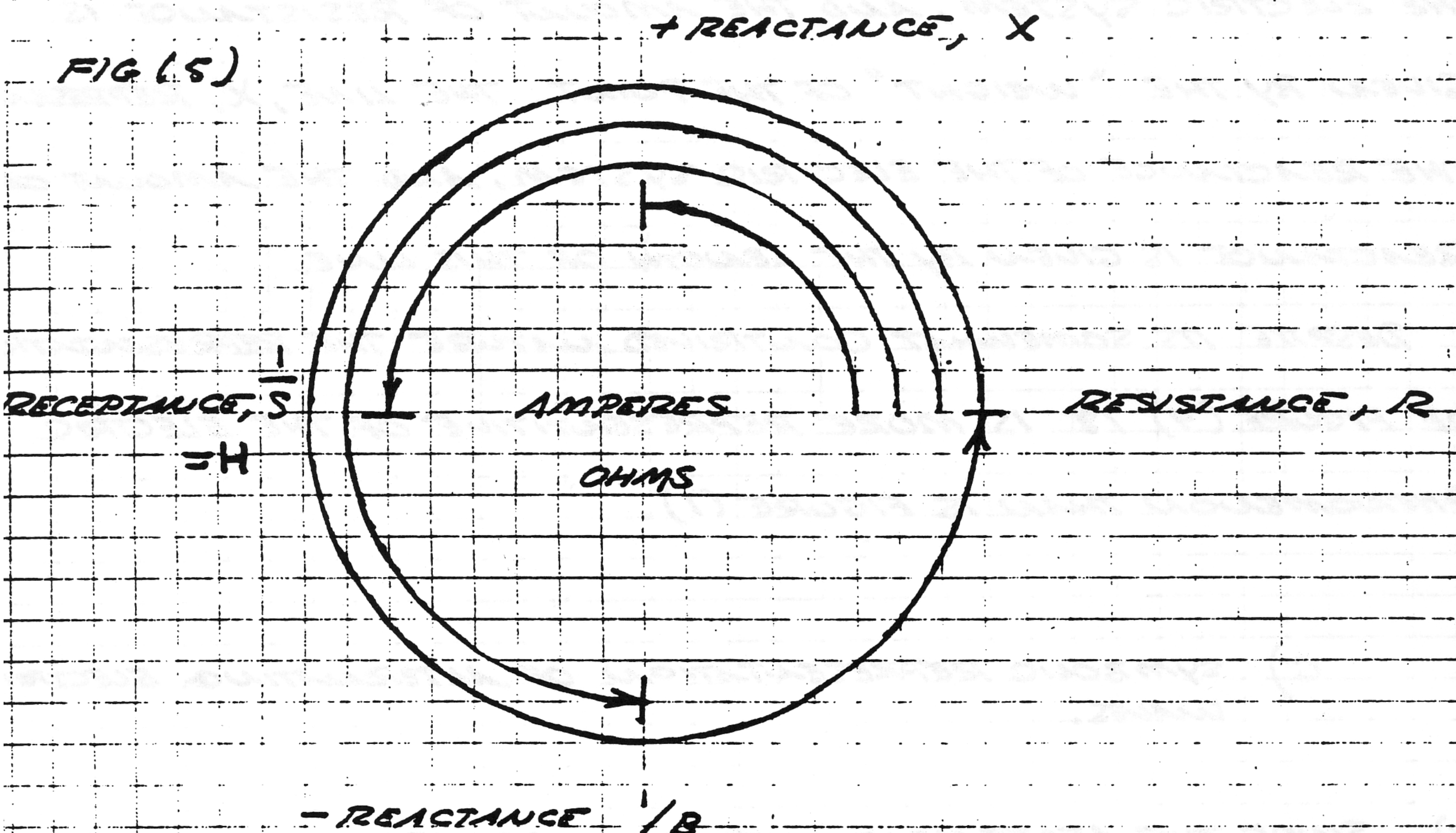
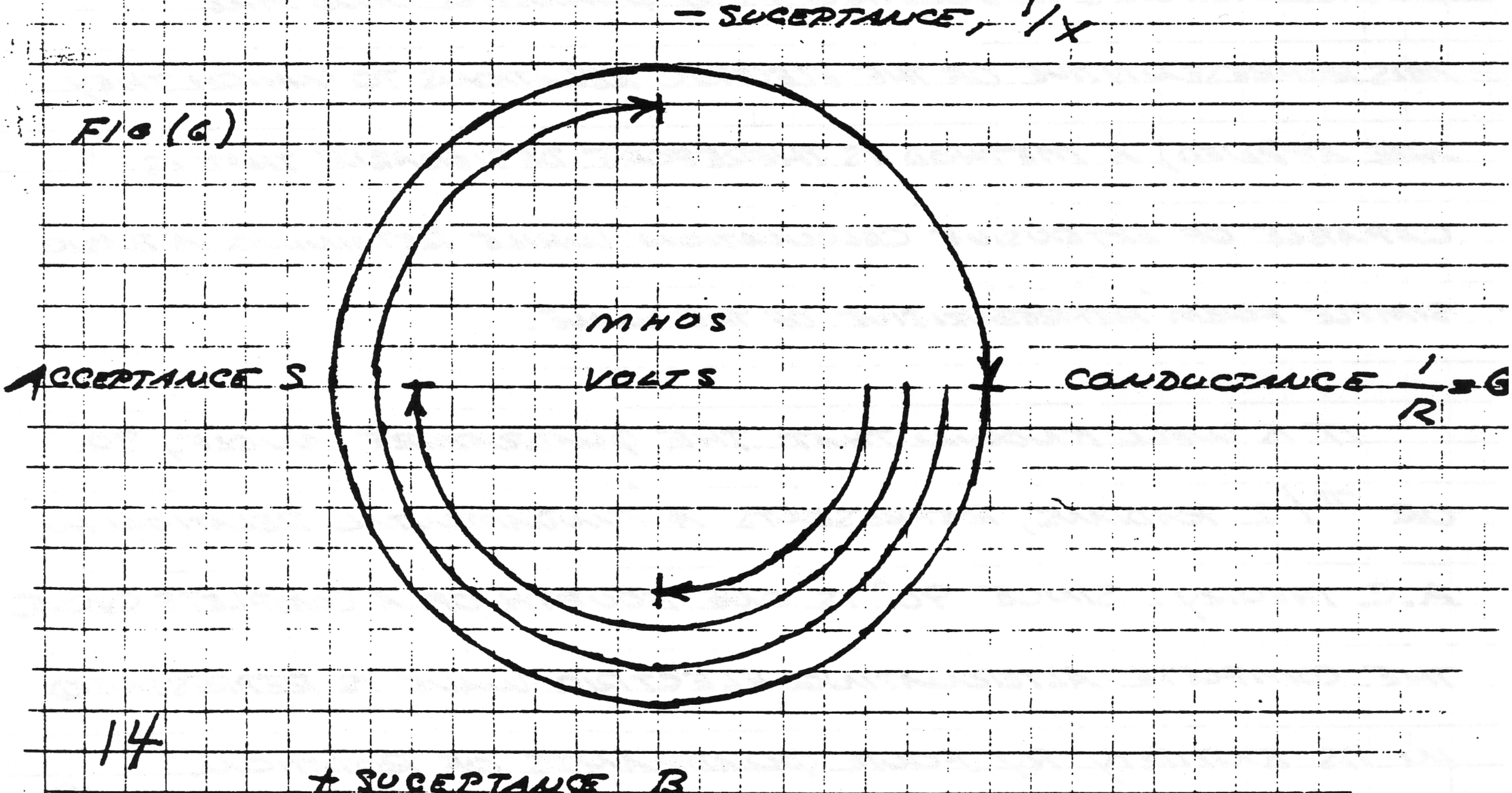
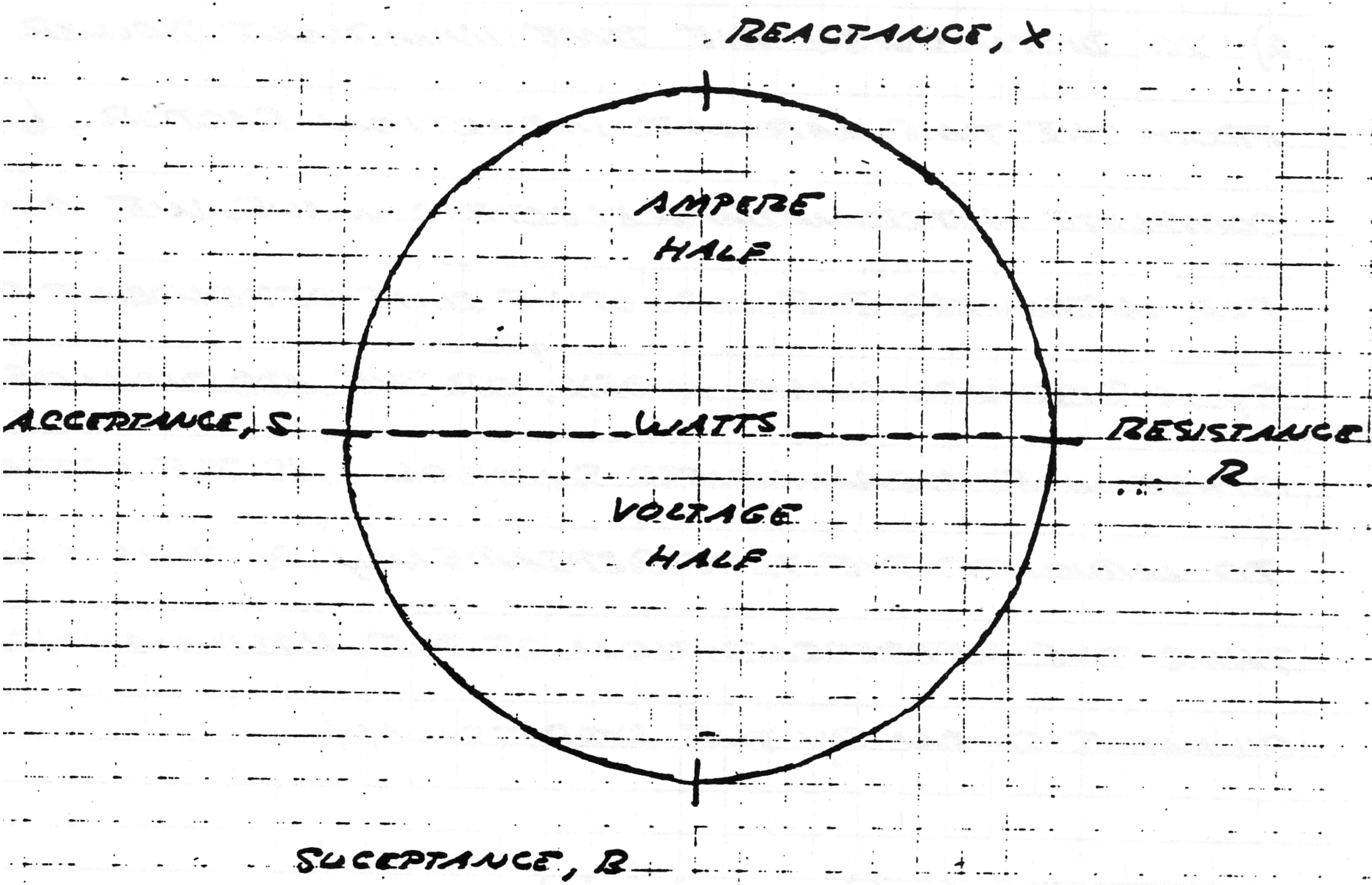


FIG (G)



− SUCEPTANCE, 1/X
MHOS
VOLTS
ACCEPTANCE S
CONDUCTANCE 1/R = G
+ SUCEPTANCE B


REACTANCE, X
AMPERE HALF
ACCEPTANCE, S
WATTS
RESISTANCE R
VOLTAGE HALF
SUCEPTANCE, B

EXPRESSED IN RECTANGULAR COÖRDINATES THE RESISTANCE-ACCEPTANCE AXIS IS EXPRESSED BY

$$\pm a = \cos\theta = \text{POWER FACTOR, PERCENT}$$

AND THE REACTANCE - SUCEPTANCE AXIS IS EXPRESSED BY

$$\pm b = \sin\theta = \text{INDUCTION FACTOR, PERCENT}$$

WHERE θ IS THE TIME POSITION OF THE ALTERNATING ELECTRIC WAVE.

2) TO DISTINGUISH THE TIME INVARIANT POWER FACTOR, a, FROM THE TIME VARIANT INDUCTION FACTOR, b, OF THE COMPLETE ALTERNATING ELECTRIC WAVE, WE MAY MARK, FOR INSTANCE, THE INDUCTIVE TIME DEPENDENT COMPONENT, BY A DISTINGUISHING INDEX[9], OR THE ADDITION OF AN OTHERWISE MEANINGLESS SYMBOL, AS THE LETTER, k, TO INDICATE THE TIME DEPENDENCY OF THIS COMPONENT. THUS THE REPRESENTATION OF THE ALTERNATING WAVE IS GIVEN IS GIVEN BY THE EXPRESSION;

$$(\sim/t^2) = a + kb \qquad \text{NUMERIC} \qquad (9)$$

WHICH HAS THE MEANING THAT THE WAVE FACTOR, (γ_t^2),

THE WAVE IS THE SUM OF THE TIME INVARIANT POWER

FACTOR OF THE WAVE, AND THE TIME VARIANT INDUCTION

FACTOR OF THE WAVE. BOTH FACTORS COMBINE INTO A

RESULTANT WAVE OF UNIT INTENSITY,

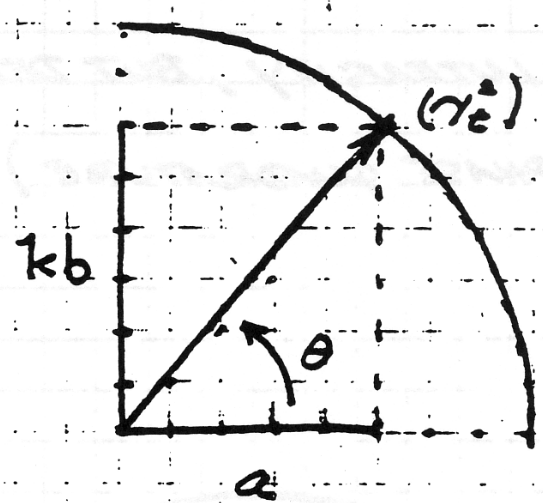
$$\left|(\gamma_t^2)\right| = \sqrt{a^2+b^2} = 1 \quad \text{UNIT RADIUS} \qquad (10)$$

AND OF TIME POSITION IN THE CYCLE OF ALTERNATION,

$$\theta = \tan^{-1} b/a \quad \text{RADIANS} \qquad (11)$$

THESE RELATIONS IN GRAPHICAL REPRESENTATION ARE GIVEN BY

FIG (8)



SIMILARILY,

$$-a-kb = -(\gamma_t^2) \quad \text{NUMERIC} \quad (12)$$

REPRESENTS A WAVE WITH THE POWER FACTOR, $-a$, AND THE INDUCTION FACTOR, $-b$, ETC.

OBVIOUSLY, THE PLUS SIGN IN THE SYMBOLIC EXPRESSION OF EQUATION (9), DOES NOT IMPLY SIMPLE ADDITION, SINCE IT CONNECTS HETEROGENEOUS QUANITIES - TIME INVARIANT & VARIANT QUANITIES, BUT IMPLIES COMBINATION AS A COMPLEX QUANITY

FOR THE PRESENT, k IS NOTHING BUT A DISTINGUISHING INDEX, AND IS OTHERWISE FREE OF DEFINITION EXCEPT THAT IT IS NOT AN ORDINARY NUMBER.

A WAVE OF UNIT INTENSITY, BUT DELAYED BY ONE QUARTER CYCLE (POSITIVE PHASE QUADRATURE) LAGS BEHIND THE WAVE $a + kb$ BY 90°

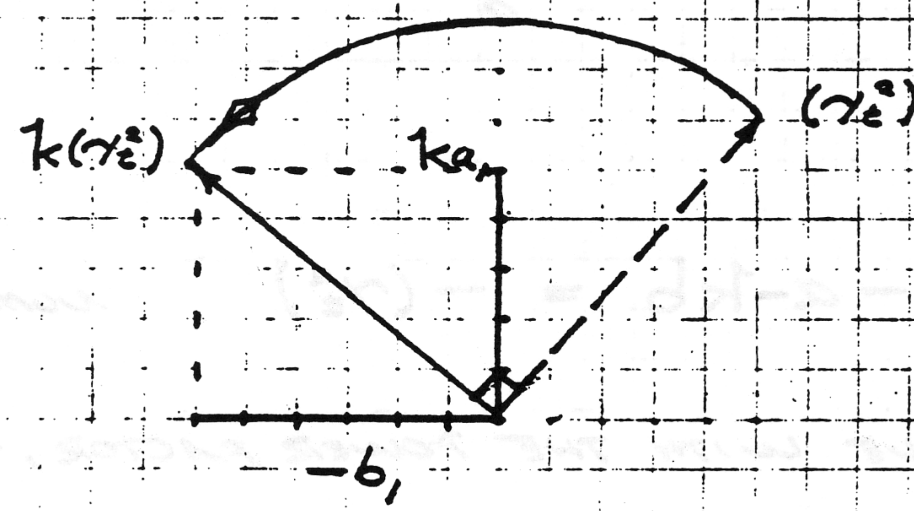


FIG (9)

THE POWER FACTOR a IS TRANSLATED INTO INDUCTION FACTOR, $k^1 a$. THE INDUCTION FACTOR b IS TRANSLATED INTO POWER FACTOR, $-b$,

THIS WAVE IS REPRESENTED SYMBOLICALLY AS;

$$k^1 a - b = k(\gamma_2^2) \qquad (13)$$

EXPLICITLY, THE ALGEBRAIC OPERATION IS GIVEN BY,

$$k(a + kb) = k^1 a + k^2 b \qquad (14)$$

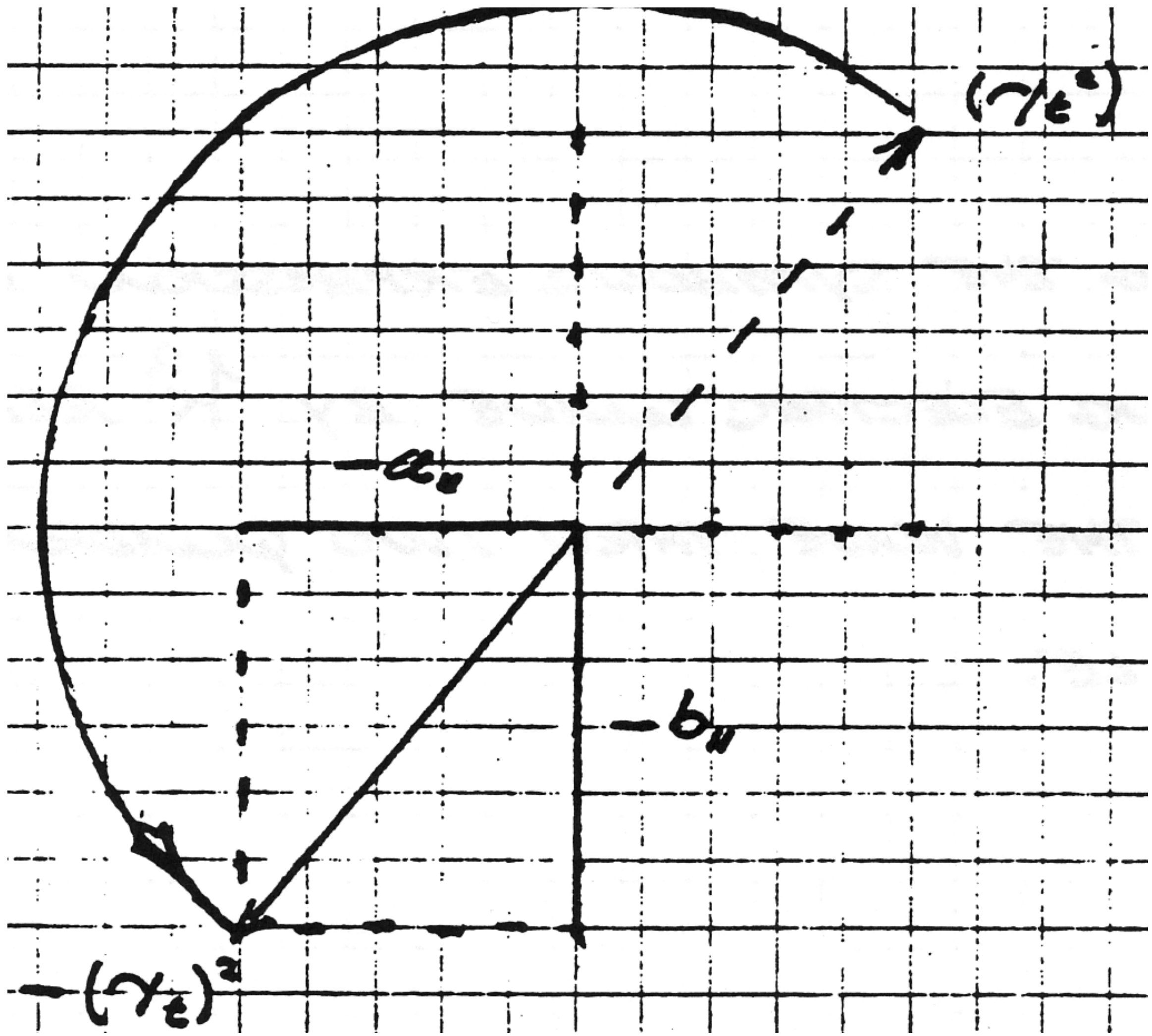
HENCE, IT IS A PROPERY OF $\mathbf{k}$ THAT,

$$\mathbf{k}^2 = -1$$

$$\mathbf{k}^1 = \mathbf{k}$$

→ MULTIPLYING THE SYMBOLIC EXPRESSION $a + kb$ OF AN ALTERNATING ELECTRIC WAVE BY k REPRESENTS THE RETARDING OF THE WAVE THRU ONE QUADRANT. THAT IS, ONE FOURTH CYCLE.

A WAVE OF UNIT INTENSITY, BUT DELAYED BY ONE HALF CYCLE (PHASE OPPOSITION) LAGS BEHIND THE WAVE $a + kb$ BY $180°$



(γ/ε^2)
$-a_{11}$
$-b_{11}$
$-(\gamma/\varepsilon)^2$

THE POWER FACTOR a AND INDUCTION FACTOR b HAVE BOTH BECOME NEGATIVE, THAT IS, REVERSED THEIR POLARITY.

THIS WAVE IS EXPRESSED SYMBOLICALLY AS

$$-a - kb = -(T/t^2) \tag{16}$$

EXPLICITLY, THE ALGEBRAIC OPERATION IS GIVEN BY,

$$k^2(a + kb) = k^2 a + k^3 b \tag{17}$$

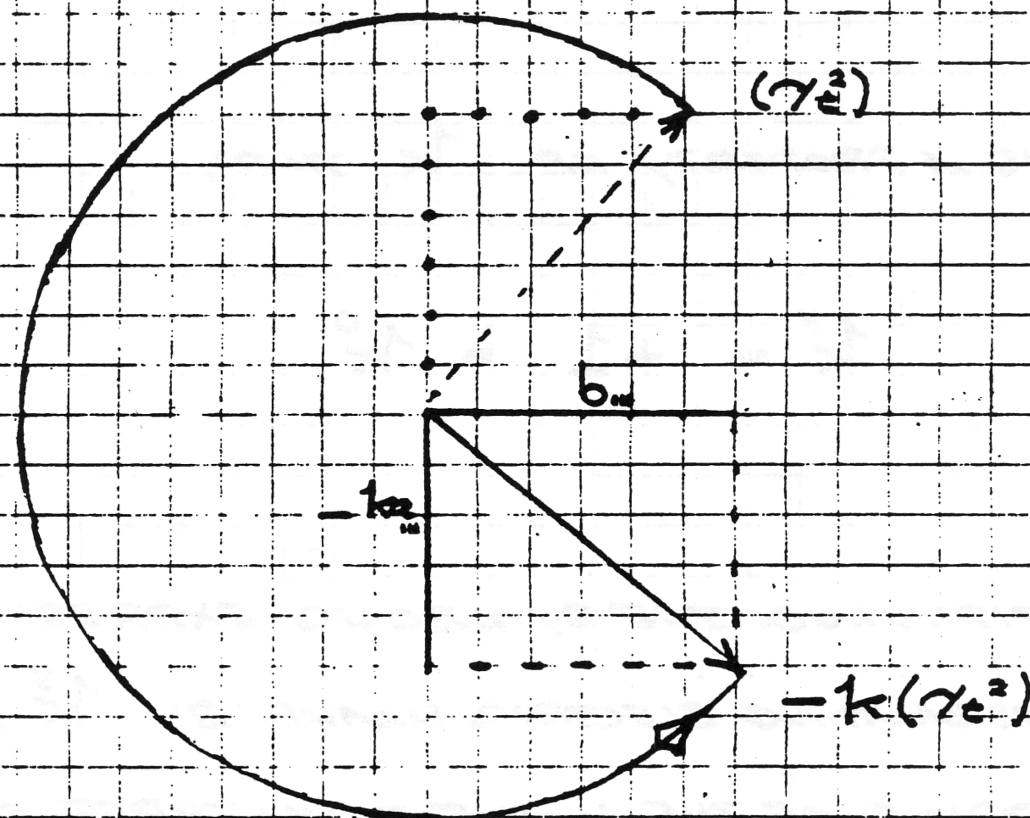
HENCE, IT IS A PROPERTY OF k THAT,

$$k^3 = -k^1 = -k \tag{18}$$

→ MULTIPLYING THE SYMBOLIC EXPRESSION $a + kb$ OF AN ALTERNATING ELECTRIC WAVE BY k^2 REPRESENTS THE RETARDING OF THE WAVE THRU TWO QUADRANTS. THAT IS, ONE HALF CYCLE.

A WAVE OF UNIT INTENSITY, BUT DELAYED BY THREE QUARTER CYCLE (NEGATIVE PHASE QUADRATURE) LAGS BEHIND THE WAVE $a + kb$ BY 270°

FIG(11)



THE POWER FACTOR, a, IS TRANSLATED INTO INDUCTION FACTOR, $-ka_m$. THE INDUCTION FACTOR, b, IS TRANSLATED INTO POWER FACTOR, b_m.

THE WAVE IS EXPRESSED SYMBOLICALLY AS

$$-ka+b = -k\left(\sim/\tfrac{a}{b}\right) \tag{19}$$

EXPLICITLY, THE ALGEBRAIC OPERATION IS GIVEN BY

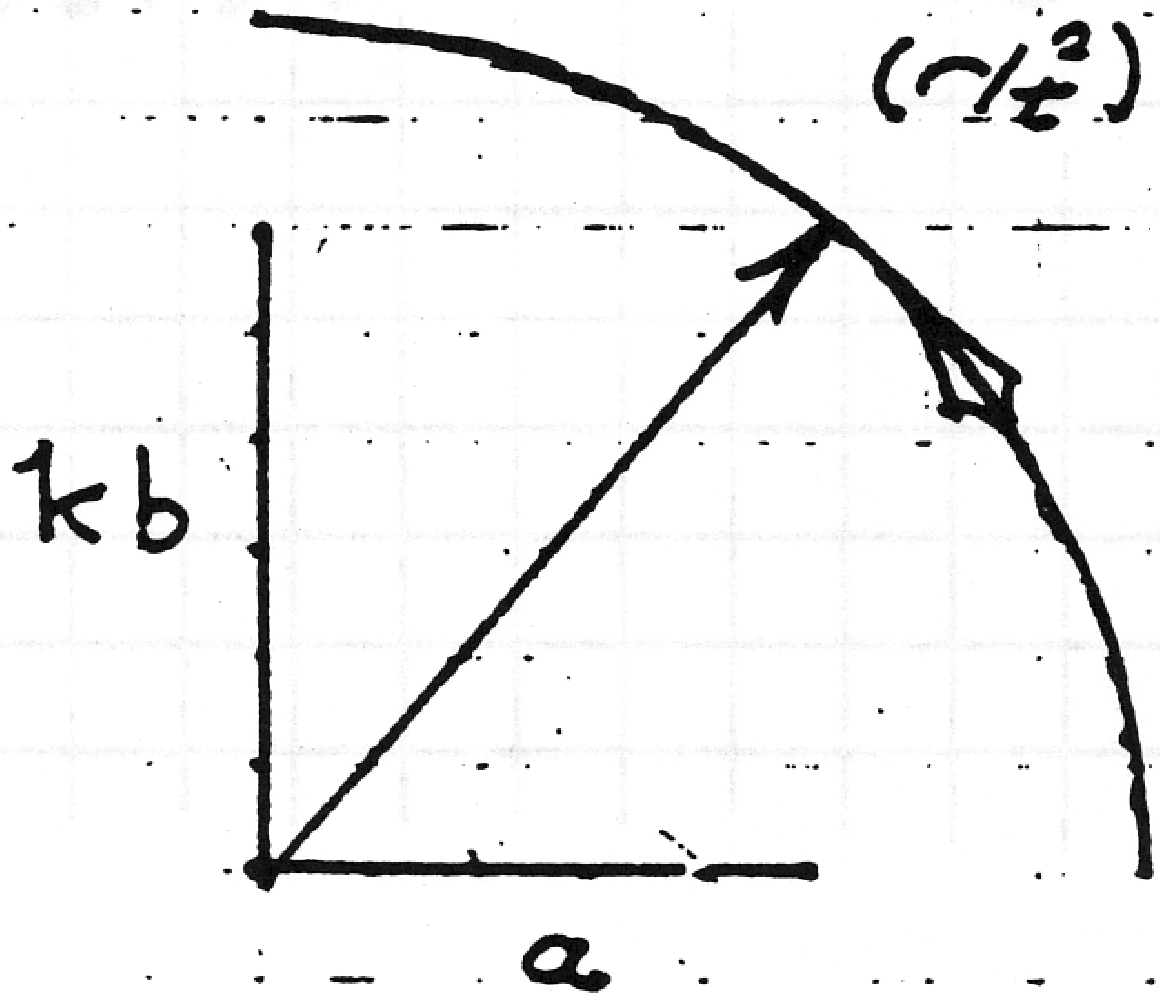
$$k^3(a+kb) = -k^1 a + k^4 b$$

HENCE, IT IS A PROPERTY OF k THAT,

$$k^4 = +1 = k^0$$

→ MULTIPLYING THE SYMBOLIC EXPRESSION $a + kb$ OF AN ALTERNATING ELECTRIC WAVE BY k^3 REPRESENTS THE RETARDING OF THE WAVE THRU THREE QUADRANTS, THAT IS, THREE QUARTER CYCLE.

A WAVE OF UNIT INTENSITY, BUT DELAYED BY A FULL CYCLE (PHASE CONJUNCTION) IS IN PHASE WITH THE WAVE $a + kb$


(r/t^2)
kb
a

THUS NO TRANSLATIONS OCCUR BETWEEN THE POWER AND INDUCTION FACTORS.

EXPLICITLY THE ALGEBRAIC OPERATION IS GIVEN BY

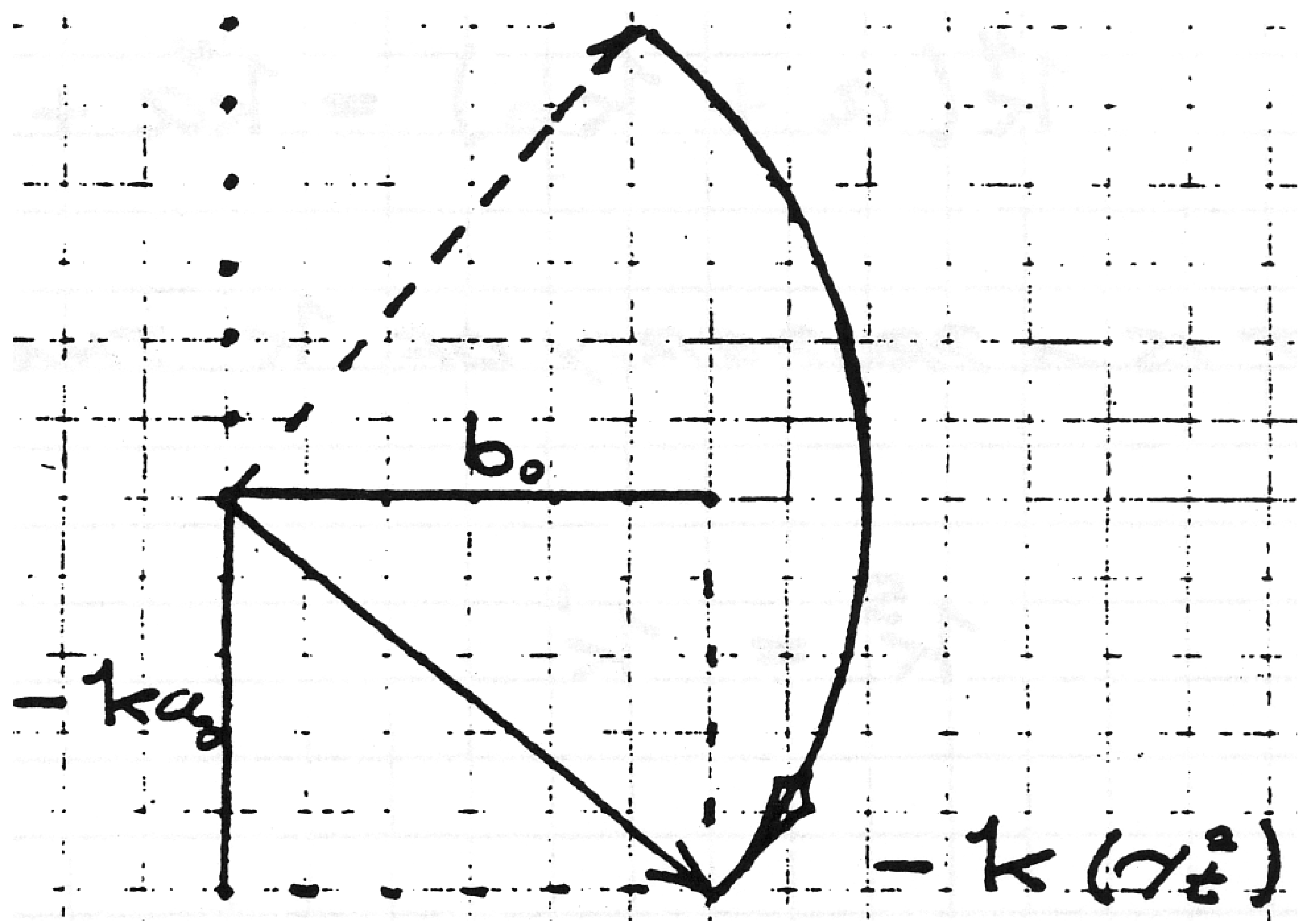
$$k^4(a + kb) = k^4 a + k^5 b \tag{22}$$

HENCE, IT IS A PROPERTY OF k THAT

$$k^5 = k^1 \tag{23}$$

MULTIPLYING THE SYMBOLIC EXPRESSION $a + kb$ OF AN ALTERNATING ELECTRIC WAVE BY k^4 REPRESENTS THE RETARDING OF THE WAVE THRU ONE FULL CYCLE & THUS LEAVES THE WAVE UNALTERED.

A WAVE OF UNIT INTENSITY, BUT ADVANCED BY ONE QUARTER CYCLE (NEGATIVE PHASE QUADRATURE) LEADS AHEAD OF THE WAVE $a + kb$ BY 90°



b_o
$-ka_o$
$-k(\gamma_t^2)$

IT IS SEEN THAT THIS IS EXACTLY THE SAME AS THE WAVE PORTRAID BY FIGURE (11), AND IS SYMBOLIZED BY EQUATION (19)

$$-ka + b = -k(r/t^2) \tag{19}$$

HOWEVER, THE EXPLICIT ALGEBRAIC OPERATION IS GIVEN BY

$$\frac{1}{k}(a + kb) = \frac{a}{k} + b \tag{24}$$

HENCE, IT IS A PROPERTY OF k THAT

$$\frac{1}{k} = k^3 = k^{-1} = -k^1 \tag{25}$$

DIVIDING THE SYMBOLIC EXPRESSION $a + kb$ OF AN ALTERNATING ELECTRIC WAVE BY k^1 REPRESENTS THE ADVANCING THE WAVE THRU ONE COMPLETE QUADRANT, THAT IS, ONE QUARTER CYCLE, AND IS DIRECTLY EQUIVILENT TO MULTIPLYING THE SYMBOLIC EXPRESSION $a + kb$ BY k^3

A WAVE OF UNIT INTENSITY, BUT ADVANCED BY ONE HALF CYCLE (PHASE OPPOSITION) LEADS AHEAD OF THE WAVE $a + kb$ BY 180°. THIS PRODUCES EXACTLY THE WAVE OF FIGURE (10), AND IS SYMBOLIZED BY EQUATION (16)

$$-a - kb = -(\gamma_2^2) \tag{20}$$

HOWEVER, THE EXPLICIT ALGEBRAIC OPERATION IS GIVEN BY

$$\frac{1}{k^2}\left(a + kb\right) = k^{-2}a + k^{-1}b \tag{26}$$

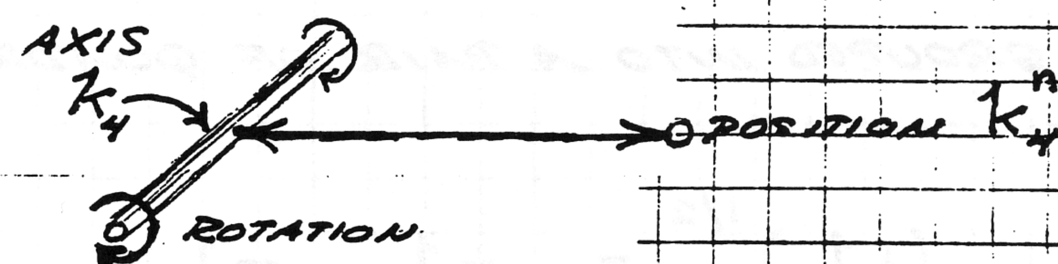
HENCE, IT IS A PROPERTY OF k THAT

$$\frac{1}{k^2} = k^2 = k^{-2} \tag{27}$$

MULTIPLYING OR DIVIDING THE SYMBOLIC EXPRESSION $a + kb$ OF AN ALTERNATING ELECTRIC WAVE BY k^2 REPRESENTS THE INVERSION OF THE WAVE, THAT IS, EITHER ADVANCING OR RETARDING THE WAVE THRU ONE HALF CYCLE, OR SIMPLY REVERSING ITS SENSE.

THEREFORE, IF WE DEFINE THE HERETOFORE MEANINGLESS SYMBOL, k, BY THE CONDITION

THE SYMBOL k^n IS A VERSOR OPERATOR WHERE k IS THE AXIS AND n IS THE AMOUNT OF TURNING AROUND THE AXIS k. SINCE THE ROTATIONAL UNIT IN THIS CASE IS $\pi/2$ OR QUARTER CYCLE THE SYMBOL IS MORE CORRECTLY GIVEN AS k_4^n.



$$k^4 = +1 \qquad (21)$$

WE ARRIVE AT

$$k^n = \sqrt[4]{+1} = 1^{1/4} \qquad (28)$$

$$n = 0, 1, 2, 3.$$

AND

$$\left.\begin{aligned} k^1 &= k \\ k^2 &= -1 \\ k^3 &= k^{-1} \\ k^0 &= +1 \end{aligned}\right\} \qquad (29)$$

THUS THE QUADRANTAL VERSOR OPERATOR k_4 SERVES AS A FUNDAMENTAL SYMBOLIC REPRESENTATION OF THE ALTERNATING ELECTRIC WAVE;

$$+(\gamma_t^2) = a + kb \qquad (30A)$$

$$-(\gamma_t^2) = -a - kb \qquad (30B)$$

HENCE

$$k_4^n = (\gamma_t^4) = k^{0,2}a + k^{1,3}b \qquad (31)$$

$$n = 0, 1, 2, 3$$

4) CHARACTERISTICS OF 1_4^n

THE ALGEBRAIC OPERATION, $1^{1/4}$, REPRESENTS A QUARTIC EQUATION AND THUS HAS FOUR DISTINCT ROOTS WHICH MAY BE GROUPED INTO A PAIR OF QUADRATICS;

$$(+1)^{1/2} = +1, -1 \qquad (32A)$$

$$(-1)^{1/2} = +j, -j \qquad (32B)$$

WHERE THE UNIT ROOT $+j$ IS OFTEN TAKEN AS THE SQUARE ROOT OF MINUS ONE WHICH IS ONLY PARTIALLY TRUE SINCE $-j$ IS ALSO A ROOT.

HENCE THE FOUR UNIT ROOTS

$$\left.\begin{array}{llll} 0) & +1 & 1) & +j \\ 2) & -1 & 3) & -j \end{array}\right\} \quad (33)$$

11

ALL FOUR ROOTS ARE IMAGINARY NUMBERS, HOWEVER THE ROOT, $+1$ IS USUALLY TAKEN AS THE REFERENCE ROOT, AND CALLED A REAL NUMBER. THESE FOUR ROOTS REPRESENT UNIT VERSORS, THAT IS, UNIT AMOUNTS OF CHANGE IN ANGULAR TIME POSITION AROUND AN AXIS k.

FOR A CONTINUING NUMBER OF CYCLES THE CHARACTERISTICS OF THE VERSOR OPERATOR k_4^n ARE GIVEN BY

TABLE (1)

$$k_4^{4m+0} = k^0 = +1 \;, \quad |+1| = 1 \;, \quad n = 0, 4, 8, 12, \ldots$$

$$k_4^{4m+1} = k^1 = +j \;, \quad |+j| = 1 \;, \quad n = 1, 5, 9, 13, \ldots$$

$$k^{4m+2} = k^2 = -1 \quad |-1| = 1 \quad n = 2, 6, 10, 14, \ldots$$

$$k^{4m+3} = k^3 = -j \quad |-j| = 1 \quad n = 3, 7, 11, 15, \ldots$$

m = Nº OF COMPLETE (360°) CYCLES OF REVOLUTION.

THESE SYMBOLS REPRESENT THE FOLLOWING ELECTRIC CONSTANTS

k^{0}, COEFFICIENT OF ENERGY CONSUMPTION;

MAGNETIC PART - RESISTANCE IN OHMS, R

DIELECTRIC PART - CONDUCTANCE IN MHOS, G

k^{1}, COEFFICIENT OF MAGNETIC ENERGY STORAGE - HENRYS PER SECOND, REACTANCE X

$\not{k}$ COEFFICIENT OF DIELECTRIC ENERGY RETURN - FARADS PER SECOND, SUCEPTANCE B

k^2, COEFFICIENT OF ENERGY PRODUCTION;

MAGNETIC PART - RECEPTANCE IN NEGATIVE OHMS H

DIELECTRIC PART - ACCEPTANCE IN NEGATIVE MHOS S

k^3, COEFFICIENT OF DIELECTRIC ENERGY STORAGE - FARADS PER SECOND, SUCEPTANCE, B

& COEFFICIENT OF MAGNETIC ENERGY RETURN - HENRYS PER SECOND, X.

THE COMPLETE EXPRESSION OF THE ALTERNATING ELECTRIC WAVE IS THUS

$$(\gamma_t^4) = \left(\overset{0}{k}a_{1} + \overset{2}{k}a_{11}\right) + \left(\overset{1}{k}b_{1} + \overset{3}{k}b_{11}\right) \qquad (34)$$

WHERE

a_1, IS THE COMPONENT OF THE POWER FACTOR REPRESENTING ENERGY CONSUMPTION

a_{11} IS THE COMPONENT OF THE POWER FACTOR REPRESENTING ENERGY PRODUCTION

b_1 IS THE COMPONENT OF THE INDUCTION FACTOR REPRESENTING ENERGY STORAGE & RETURN

b_{11} IS THE COMPONENT OF THE INDUCTION FACTOR

REPRESENTING ENERGY RETURN & STORAGE.

TABLE (2)

$$+\,{}^{1}k_{4} = +1$$

$$+\,{}^{1}k_{4}^{2} = -1$$

$$+\,{}^{1}k_{4}^{3} = -1$$

$$+\,{}^{1}k_{4}^{4} = +1$$

$$-k_4^1 = -j$$

$$-k_4^2 = +1$$

?

$$-k_4^3 = +j$$

$$-k_4^4 = -1$$

FORWARD ROTATION

$$+1k_y^{-1} = 1k_y^{3} = -j$$

$$+1k_y^{-2} = 1k_y^{2} = -1$$

$$+1k_y^{-3} = 1k_y^{1} = +j$$

$$+1k_y^{-4} = 1k_y^{0} = +1$$

$$-k_y = k_y = +j$$

$$-k_y^{-2} = k_y^{0} = +1$$

$$-k_y^{-3} = k_y^{3} = -j$$

$$-k_y^{-4} = k_y^{2} = -1$$

REVERSE ROTATION

TABLE (3)

$$(-k)^4 = +1 \qquad (-k)^3 = +j$$

$$(-k)^2 = -1 \qquad (-k)^1 = -j$$

5) TRIGONOMETRIC & EXPONENTIAL (NATURAL) EQUIVALENTS

IN TRIGONOMETRIC FORM THE VERSOR OPERATOR k_4^n IS GIVEN BY THE FOLLOWING RELATIONS.

$$\left.\begin{aligned} k_4^n &= 1^{1/4} \\ &= (+1)^{1/2} = +\cos n_0 \;,\; -\cos n_0 \\ \text{AND} \quad &= (-1)^{1/2} = +\sin n_0 \;,\; -\sin n_0 \end{aligned}\right\} \quad (35)$$

WHERE $\quad n_0 = \dfrac{\pi}{2} n$

HENCE

$$\left.\begin{aligned} k_4^n = \big(& k^0 \cos n_0 + k^1 \sin n_0 \\ & + k^2 \cos n_0 + k^3 \sin n_0 \big) \end{aligned}\right\} \quad (36)$$

SUBSTITUTING EQUATION (34) INTO (36) GIVES

$$k^0 \cos n_0 = +\cos n_0 = +a_I \quad \text{ENERGY CONSUMPTION}$$

$$k^2 \cos n_0 = +\cos n_0 = -a_{II} \quad \text{ENERGY PRODUCTION}$$

$$k^1 \sin n_0 = +\sin n_0 = +b_I \quad \text{ENERGY STORAGE / RETURN}$$

$$k^3 \sin n_0 = -\sin n_0 = -b_{II} \quad \text{ENERGY RETURN / STORAGE}$$

IT CAN BE SEEN THAT ENERGY CONSUMPTION & PRODUCTION IS REPRESENTED BY THE EVEN FUNCTION, ENERGY STORAGE & RETURN BY THE ODD FUNCTION, HENCE

$$
\left.
\begin{aligned}
&a)\quad +\cos n_0 = +1 - \frac{n_0^2}{2!} + \frac{n_0^4}{4!} - \frac{n_0^6}{6!} + , \ldots \\
&2)\quad -\cos n_0 = -1 + \frac{n_0}{2!} - \frac{n_0}{4!} + \frac{n_0}{6!} - , \ldots \\
&1)\quad +\sin n_0 = +n_0 - \frac{n_0}{3!} + \frac{n_0}{5!} - \frac{n_0}{7!} + , \ldots \\
&3)\quad -\sin n_0 = -n_0 + \frac{n_0}{3!} - \frac{n_0}{5!} + \frac{n_0}{7!} - , \ldots
\end{aligned}
\right\} \quad (37)
$$

SUBSTITUTING INTO EQUATION (3) THE EXPONENTIAL EQUATIONS

$$\pm \cos n_0 = \pm \frac{1}{2}\left[\epsilon^{+jn_0} + \epsilon^{-jn_0} \right] \qquad (38)$$

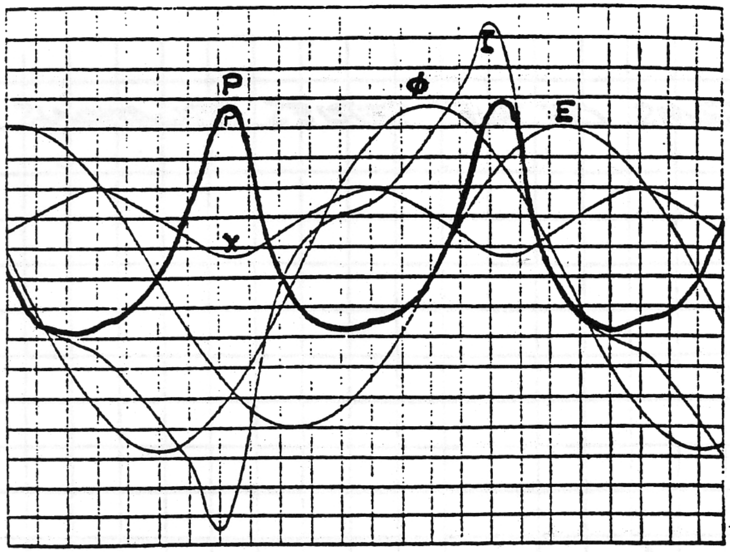
$$\pm \sin n_0 = \pm \frac{1}{2}\left[\epsilon^{-jn_0} - \epsilon^{+jn_0} \right] \qquad (39)$$

GIVES

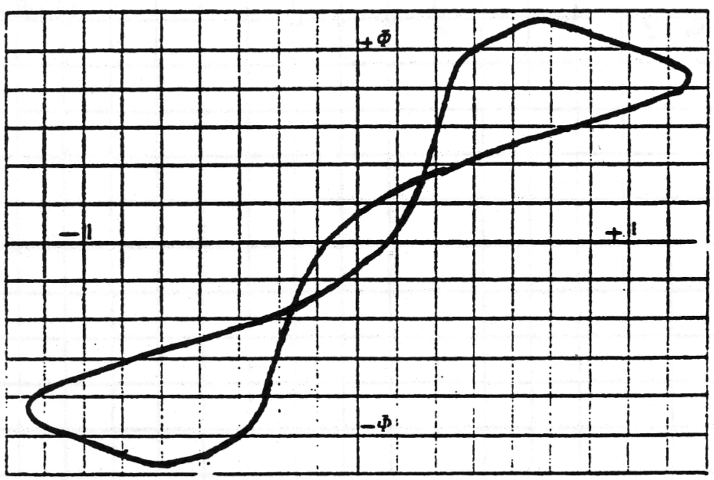
$$k_y^n = \epsilon^{\pm j n_0} = \epsilon^{\sqrt[2]{1}\, n_0} \qquad (40)$$

THUS THE VERSOR OPERATOR K_4^n ALSO SERVES AS THE BASIS OF IMAGINARY LOGARITHMS AND ELIMINATES THE NECESSITY OF UTILIZING THE SQUARE ROOT OF MINUS ONE IN THE EXPONENT WHEN EXPRESSING AN ALTERNATING ELECTRIC WAVE IN EXPONENTIAL FORM.

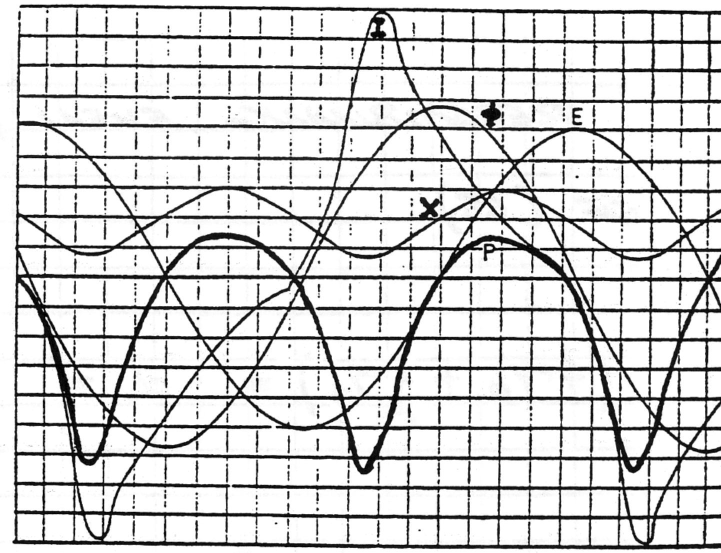
IT IS THEN ALSO POSSIBLE FOR n TO BE OF NON INTEGER VALUE ALLOWING FOR THE EXPRESSION OF HERETOFORE UNEXPLORED ELECTRIC WAVES.



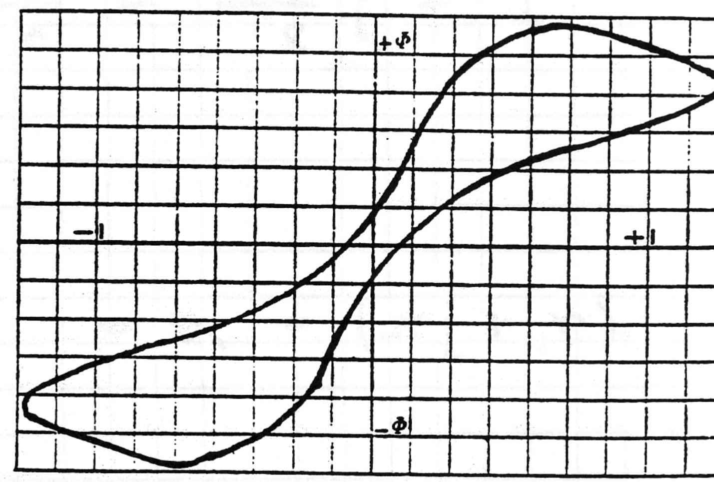
A *Variable Reactance, Reaction Machine.*



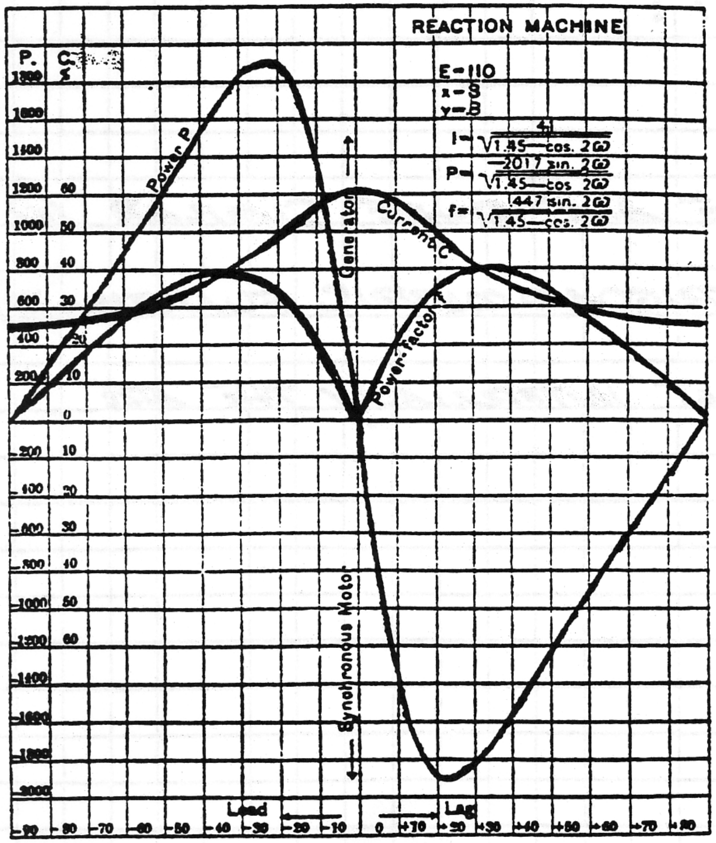
B *Hysteretic Loop of Reaction Machine.*



C *Variable Reactance, Reaction Machine.*



D *Hysteretic Loop of Reaction Machine.*



E

Reaction Machine.

ROTATING APPARATUS EXHIBITING CANONIC ELECTRIC WAVES

FIGURES (A) & (B): PRODUCTION OF ELECTRIC ENERGY

FIGURES (C) & (D): CONSUMPTION OF ELECTRIC ENERGY

FIGURE (E): COMPOSITE

6) CANONIC ELECTRIC WAVES

a) FROM THE PROCEEDING SECTIONS IT CAN BE CONCLUDED THAT THE GENERALIZED VERSOR OPERATOR IS GIVEN AS

$$k_N^n = 1^{1/N} \tag{70}$$

WHERE

k IS THE AXIS OF ROTATION

N IS THE NUMBER OF UNIT DIVISIONS

n IS THE SPECIFIC AMOUND OF VARIATION

IF THE NUMBER OF DIVISIONS N IS A POWER OF 2 THE WAVE CAN BE EXPRESSED IN TERMS OF LOG BASE ε OTHERWISE A NEW BASIS OF LOGARITHMS IS REQUIRED.

IN QUADRATURE TRIGONOMETRIC FORM THE GENERALIZED OPERATOR IS OF THE FORM

$$k_N^n = k_4^{0,2} \cos 2\pi \frac{n}{N} + k^{1,3} \sin 2\pi \frac{n}{N} \tag{71}$$

THE OPERATOR k_N^n REPRESENTS THE DIVISION OF THE ALTERNATING WAVE INTO N UNITS OF VARIATION THRU THE CYCLE. THUS THE GENERALIZED SYMBOLIC EXPRESSION OF THE ELECTRIC WAVE IS DIVIDED INTO N FACTORS,

$$(\gamma_t^N) = A\,k_N^0 + B\,k_N^1 + \ldots (\text{ETC})\,k_N^{N-1} \qquad (72)$$

b) SINCE THE ALTERNATING ELECTRIC WAVE IS CHARACTERISTICALLY OF QUADRANTAL, OR FOUR POLE, FORM, k_4^n, THE FOUR CHARACTERISTICS BASICALLY BEING,

0)	RESISTANCE, OHMS	+1	R	
1)	REACTANCE, HENRYS / SEC	+j	X	TABLE (5)
2)	ACCEPTANCE, MHOS	-1	S	
3)	SUCEPTANCE, FARADS/SEC	-j	B	

THEN ESTABLISHING ELECTRIC WAVES IN SYSTEMS OF ANGULAR DIVISION OTHER THAN QUADRANTAL, SUCH AS TRIPLE PHASE, PRODUCES A TYPE OF INTERFERENCE PATTERN BETWEEN THE NATURAL FORM OF THE WAVE AND THE FORM IMPOSED ON IT

AN IMPORTANT CLASS OF ELECTRIC WAVE IS ONE IN WHICH THE QUANITY

$$(XG - RB) - (XS - HB)$$

IS NON ZERO. THIS CONDITION RESULTS FROM THE RATE OF ENERGY CONSUMPTION DIFFERING FROM THE RATE OF ENERGY PRODUCTION, DISTORTING THE WAVEFORM. THIS PRODUCES ELECTRIC WAVES THAT GROW OR DECAY WITH RESPECT TO TIME. SUCH WAVES ARE TRANSIENT ELECTRIC WAVES. THESE WAVES A CHARACTERIZED BY HAVING A FREQUENCY OR PERIOD THAT IS A COMPLEX QUANITY CONSISTING OF REAL AND IMAGINARY COMPONENTS.

$$\dot{\nu} = (\omega - j\mu) \quad \text{NEPER}\cdot\text{RADIANS / SEC} \tag{83}$$

THE REAL COMPONENT OF THE COMPLEX FREQUENCY, ω, IN RADIANS PER SECOND REPRESENTS THE CYCLIC PERIOD OF REVOLUTION IN WHICH THE WAVE REPEATS THE SAME MINIMUM VALUE OF AMPLITUDE IN EQUAL TIME INTERVALS. THE IMAGINARY COMPONENT, μ, IN NEPERS PER SECOND REPRESENTS THE ACYCLIC PERIOD OF EVOLUTION IN WHICH THE MAXIMUM VALUE OF AMPLITUDE INCREASES OR DECREASES AT A CONSTANT GEOMETRIC RATE.

TRANSIENT ELECTRIC WAVES MAY BE DIVIDED INTO TWO CATEGORIES, THOSE WAVES WHICH REPEAT THE MINIMUM VALUE OF AMPLITUDE IN EQUAL INTERVALS OF TIME, AND THOSE WAVES WHICH DO NOT REPEAT ANY VALUE OF AMPLITUDE MORE THAN ONCE.

THE FORMER CATEGORY OF WAVE IS CALLED AN OSCILLATING ELECTRIC WAVE. THIS WAVE IS CHARACTERIZED BY THE CONDITION

$$|a_0| > |b_0| \tag{84}$$

THE LATTER CATEGORY IS CALLED AN ELECTRIC IMPULSE AND IS CHARACTERIZED BY THE CONDITION

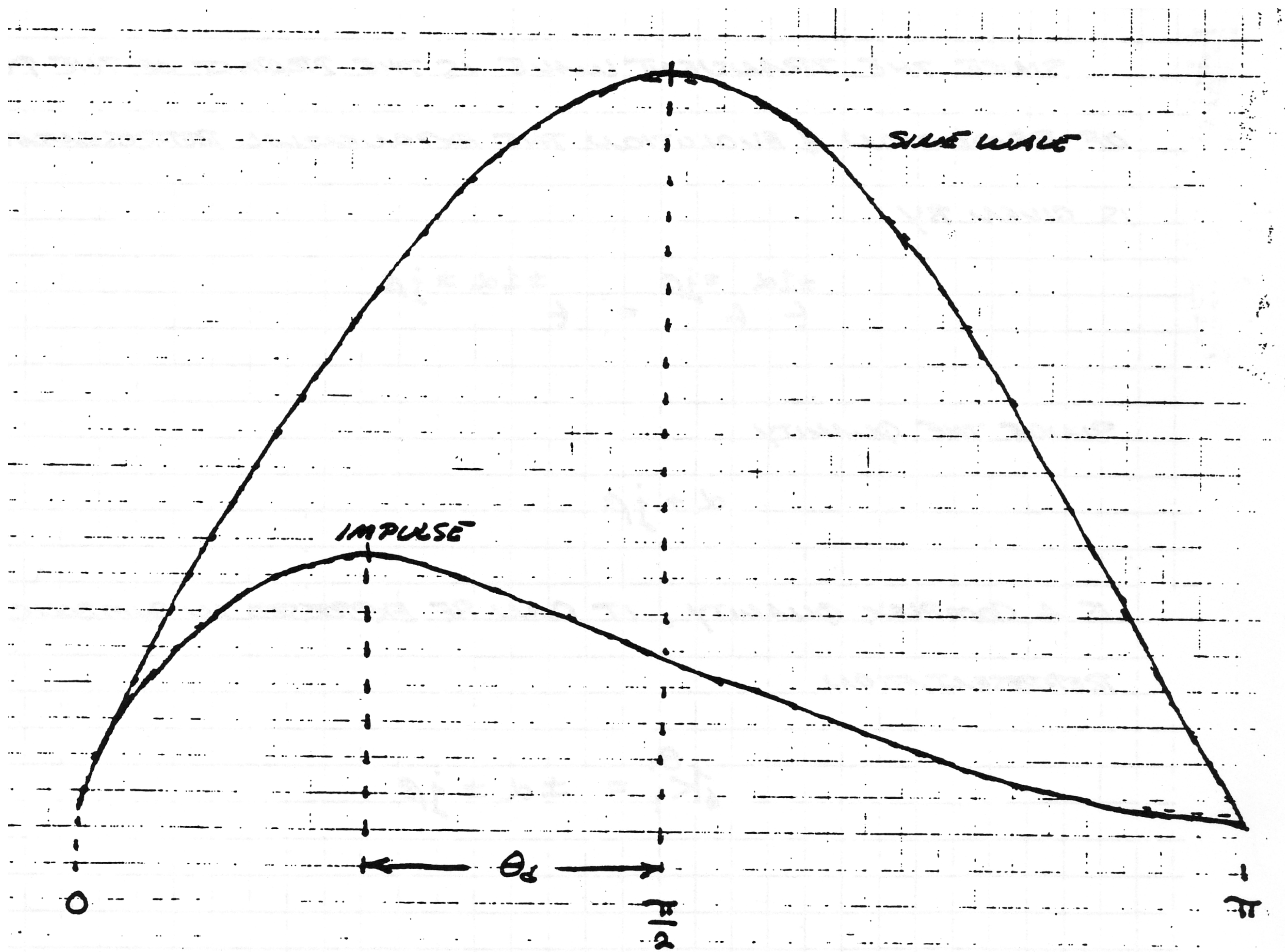
$$|a_0| < |b_0| \tag{85}$$

A PARTICULAR CHARACTERISTIC OF THE TRANSIENT ELECTRIC WAVE IS THE DISPLACEMENT OF THE MAXIMUM VALUE OF THE WAVE FROM THE POINT OF MAXIMUM OF AN EQUIVALENT ALTERNATING WAVE AS SHOWN IN FIG ( ). THE ANGLE OF DISPLACEMENT IS GIVEN BY

$$\theta_d = TAN^{-1} \frac{b_0}{a_0} \qquad (86)$$

$\theta_d > 45^\circ$, OSCILLATION

$\theta_d < 45^\circ$, IMPULSE



SINE WAVE
IMPULSE
θ_d
0
$\frac{\pi}{2}$
π

e) THE EXPONENTIAL REPRESENTATION OF THE ALTERNATING ELECTRIC WAVE IS GIVEN BY

$$\epsilon^{\pm j\beta} = \cos\beta \pm j\sin\beta \tag{87}$$

WHERE THE ANGLE β REPRESENTS THE TIME POSITION OF REVOLUTION.

ANALOGOUSLY, THE WAVE OF GEOMETRIC PROGRESSION IS GIVEN BY

$$\epsilon^{\pm 1\alpha} = \cosh\alpha \mp j^2 \sinh\alpha \tag{88}$$

WHERE THE ANGLE α REPRESENTS THE TIME POSITION OF EVOLUTION.

SINCE THE TRANSIENT WAVE IS THE PRODUCT OF THE PERIODS OF REVOLUTION & EVOLUTION THE EXPONENTIAL REPRESENTATION IS GIVEN BY

$$\epsilon^{\pm 1\alpha}\,\epsilon^{\pm j\beta} = \epsilon^{\pm 1\alpha \pm j\beta} \tag{89}$$

SINCE THE QUANITY

$$\alpha \pm j\beta$$

IS A COMPLEX QUANITY, IT CAN BE EXPRESSED IN SYMBOLIC REPRESENTATION

$${}_{1}K_{1}^{\theta_1} = \pm\alpha \pm j\beta \tag{90}$$

WHERE THE SUBSCRIPT 1 IN k_1 DOES NOT INDICATE THE BASE, SINCE BASE FOUR IS ASSUMED, BUT HOWEVER DISTINGUISHES THE AXIS FROM THAT OF k IN PREVIOUS CALCULATIONS. SUBSTITUTING THE RELATION

$$\epsilon^{j\frac{\pi}{2}} = k^1$$

AND

$$n_{10} = \frac{\pi}{2} n_1$$

$$n_0 = \frac{\pi}{2} n$$

INTO (89) AND (40) GIVES

$$\epsilon^{(\pm\alpha \pm j\beta) n_0} = \epsilon^{k_1^{\theta_1} n_0} = k^{n\, k_1^{n_1}} \qquad (91)$$

WHERE BOTH VERSORS ARE ASSUMED BASE FOUR (k_4).

EQUATION (91) IS CALLED A HYPERCOMPLEX QUANITY IN THAT IT POSSESSES A COMPLEX QUANITY WITHIN A COMPLEX QUANITY. OBVIOUSLY THIS COULD BE CARRIED INDEFINITLY

$$k^{n k_1^{n_1 k_{11}^{n_{11}} \cdots}}$$

PRODUCING EXCEEDINGLY COMPLEX WAVEFORMS.

THIS EQUATION (91) SERVES AS A BASIC SYMBOLIC EXPRESSION OF THE GENERALIZED TRANSIENT ELECTRIC WAVE FOR THE CONDITION OF DIRECT CORRELATION BETWEEN CAUSE & EFFECT. THIS REPRESENTATION INDICATES VARIATION WITHIN VARIATION OF THE WAVE.

SUBSTITUTING

$$k_1^{n_1} = k_1^{0,2} \cos n_{1a} + k_1^{1,3} \sin n_a$$

INTO EQUATION (91) GIVES

$$k^{n\left(k_1^{0,2}\cos n_{10} + k^{1,3}\sin n_{10}\right)} \qquad (92)$$

$$= k^{n\, k_1^{0,2}\cos n_{10}} \times k^{n\, k_1^{1,3}\sin n_{10}} \qquad (93)$$

AND

$$k^{n\, k_1^{0,2}\cos n_{10}} =$$

$$k^{0,2}\cos\left(n_{10}\, k^{0,2}\cos n_{10}\right) + k^{1,3}\sin\left(n_{0}\, k^{0,2}\cos n_{10}\right) \qquad (94)$$

$$\overline{k^{n} k_1^{1,3} \sin n_{10}} =$$

$$k^{0,2} \cosh\left(n_0 k_1^{0,2} \sin n_{10}\right) + k^{1,3} \sinh\left(n_0 k_1^{0,2} \sin n_{10}\right) \tag{95}$$

SUBSTITUTING $\cos n_{10} = a_1$

$\sin n_{10} = b_1$

AND COMBINING (94) & (95)
GIVES

$$+1\left[\cos a_1 n_{10}\ \cosh b_1 n_{10} - \sin a_1 n_{10}\ \sinh b_1 n_{10}\right]$$

(96)

$$+j\left[\cos a_1 n_{10}\ \sinh b_1 n_{10} - \sin a_1 n_{10}\ \cosh b_1 n_{10}\right]$$

SUBSTITUTING $a_1 n_{10} = \theta_a$ $b_1 n_{10} = \theta_b$

GIVES

$$\left[\cosh\theta_b\left(\cos\theta_a - j\sin\theta_a\right)\right] \pm j\left[\sinh\theta_b\left(\cos\theta_a - j\sin\theta_a\right)\right]$$

(97)

AND SUBSTITUTING

$$K_o^{\theta} = \cos\theta_a - j \sin\theta_a$$

GIVES

$$K_o^{\theta}\left[\cosh\theta_b + j\sinh\theta_b\right] \tag{98}$$

AND PUTTING

$$K_o^{\theta} \cosh\theta_b = A$$

$$K_o^{\theta} \sinh\theta_b = B$$

HENCE

$$(\dot{\gamma}_t^{4}) = A + jB \tag{99}$$

AND THUS THE SYMBOLIC EXPRESSION OF THE GENERALIZED ELECTRIC WAVE,

$$k^{n}\,k_1^{n_1}, \qquad k^{n}\text{, PRIMARY QUADRANTAL VERSOR.}$$

$$k_1^{n_1}\text{, SECONDARY QUADRANTAL VERSOR.}$$

n, TIME ANGLE (IN QUADRANTS) OF POSITION ALONG WAVE,

n_1, TIME ANGLE (IN QUADRANTS) OF PHASE DISTORTION OF THE WAVE, A FUNCTION Θd

(IV) INDUCTION IN THE DIMENSION OF SPACE

a) PRODUCT OF CONJUGATE PAIR OF INDUCTIONS

THE WAVE THEORIES IN PRESENT USAGE FOR THE STUDY OF ELECTRIC PROPAGATION ALONG COILS AND KINDRID APPARATUS ALL SUFFER FROM THE FUNDAMENTAL DRAWBACK THAT THEY ARE REPRESENTATIONS OF ENERGY PROPAGATION ALONG A SINGLE LINE OR AXIS. THE EQUIVALENT CIRCUIT OF COIL PROPAGATION IS, HOWEVER BEST REPRESENTED AS IN FIGURE (1), THAT IS, TWO PERPENDICULAR PATHS FOR INDUCTION. THUS THE PROPAGATION CAN OCCUR IN ANY DIRECTION ON THE SURFACE OF THE MESH GIVEN BY FIGURE (1).

THE NATURE OF ELECTRIC ENERGY VARIES WITH THE DIRECTION OF PROPAGATION AND DEPARTS SIGNIFICANTLY FROM THE COMMON ELECTRO-MAGNETIC FORM WHEN THE PATH IS NO LONGER ALONG THE USUAL AXIS. THIS DEPARTURE IN FORM IS OF SINGULAR IMPORTANCE IN THE STUDY OF TESLA'S DISCOVERIES.

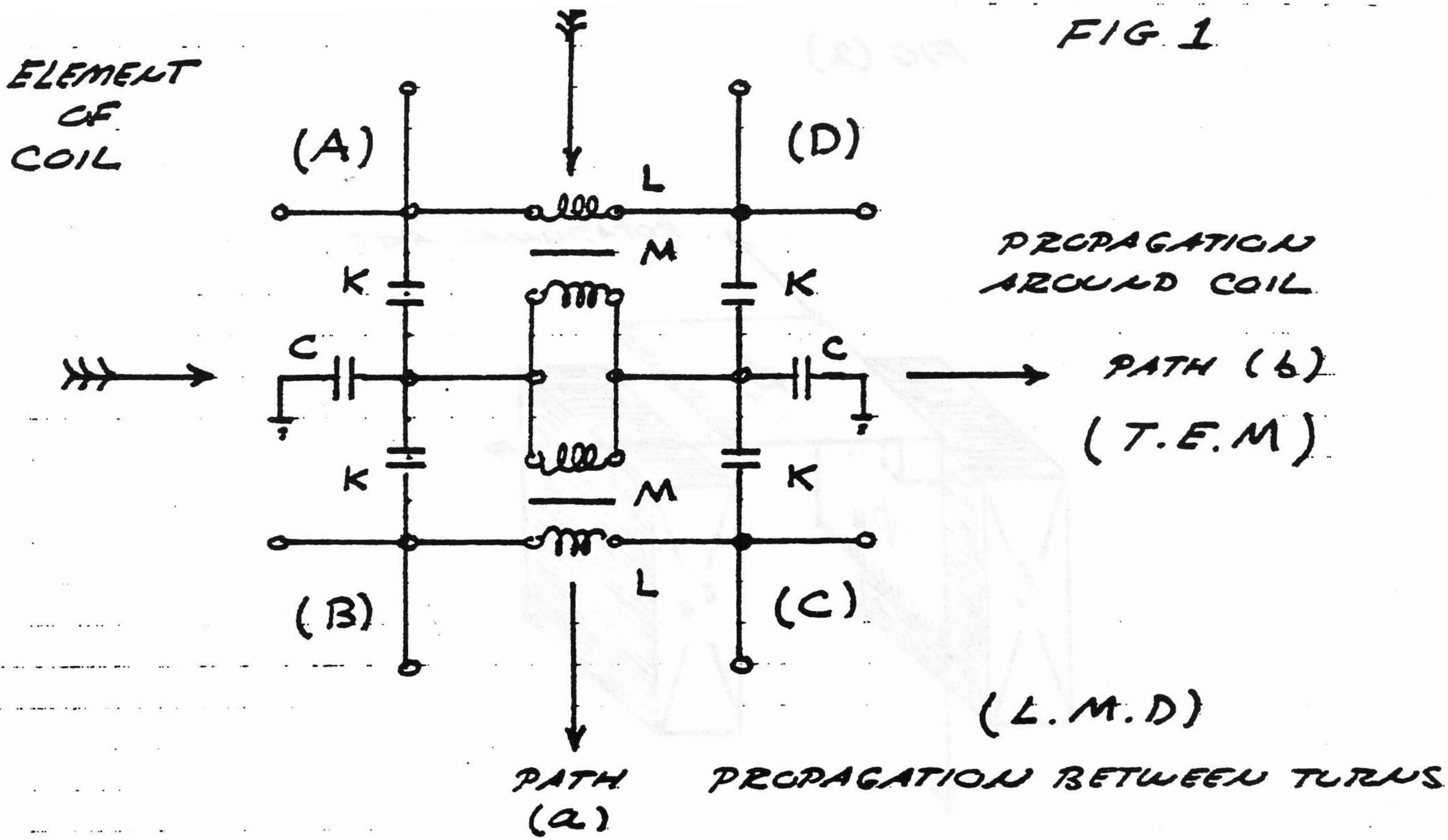
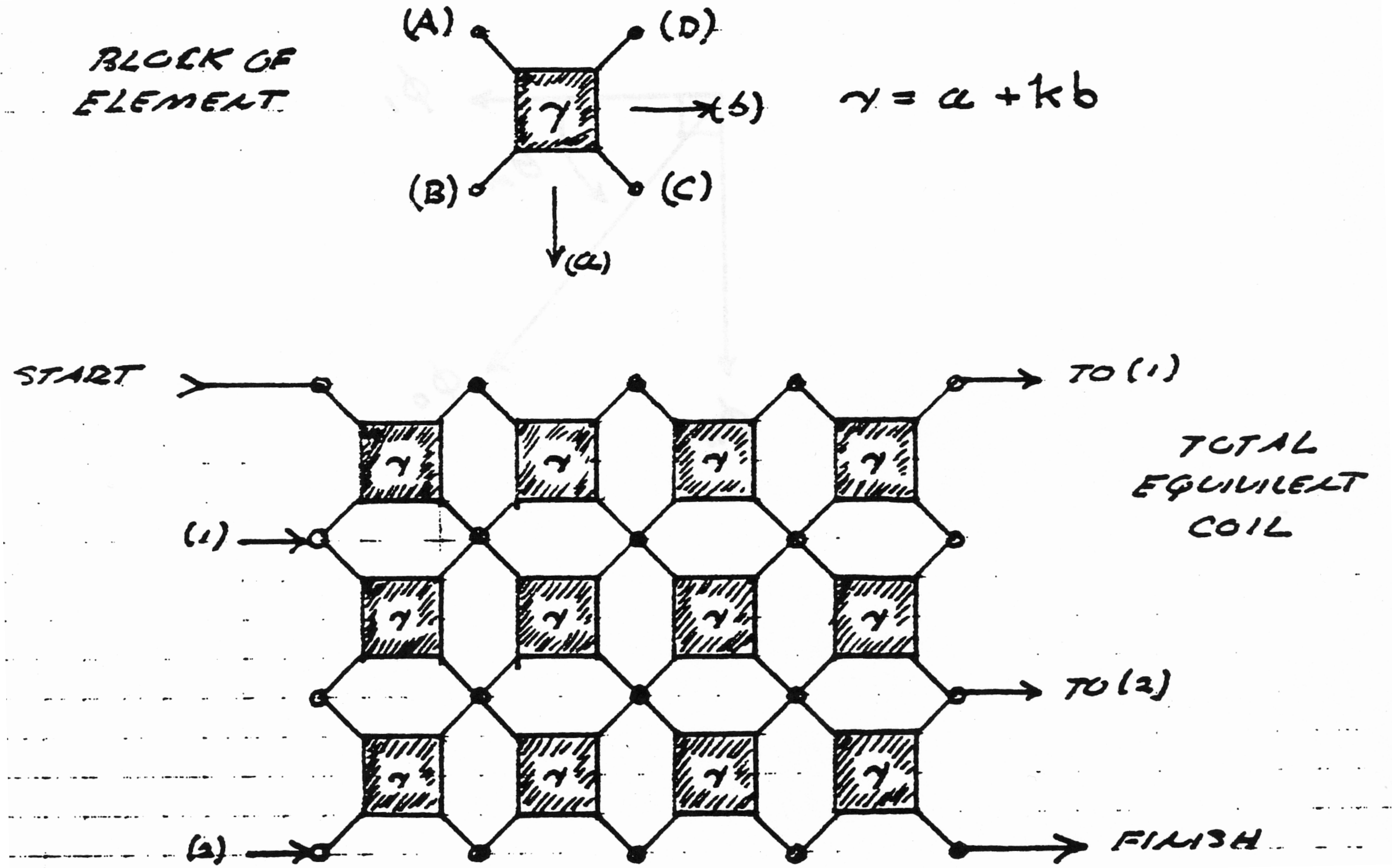
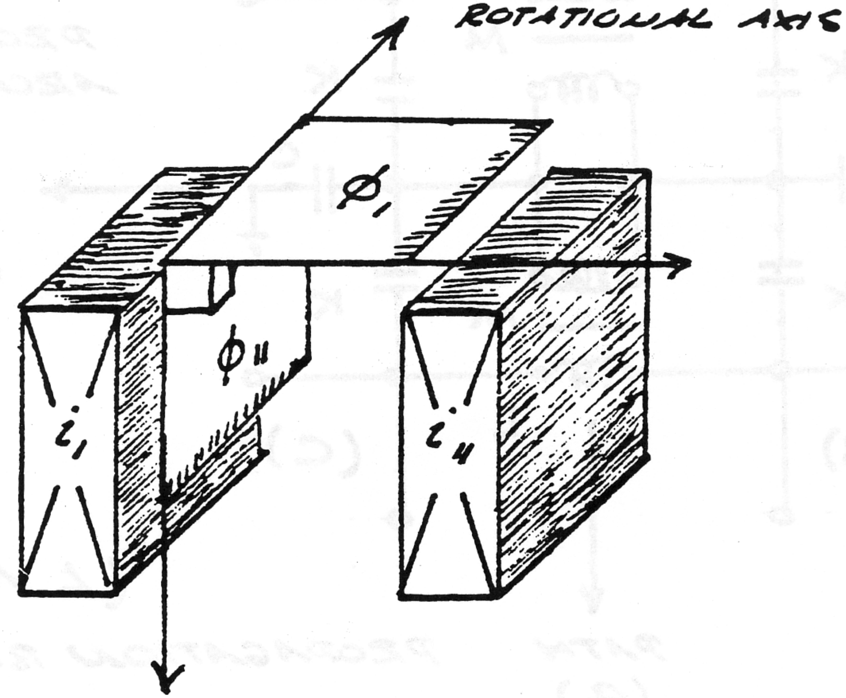
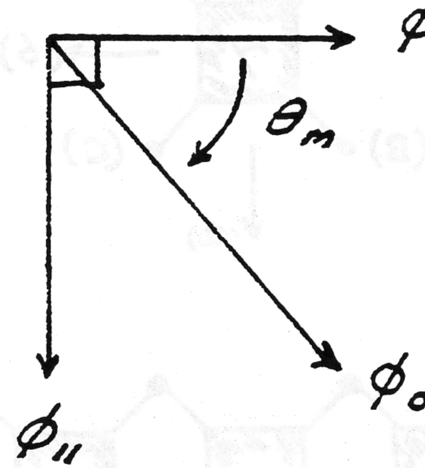

ELEMENT OF COIL
(A)
(D)
L
M
K
K
C
C
PROPAGATION AROUND COIL
PATH (b)
(T.E.M)
K
K
M
L
(B)
(C)
(L.M.D)
PATH (a)
PROPAGATION BETWEEN TURNS

FIG 1


BLOCK OF ELEMENT
(A)
(D)
γ
(b)
(B)
(C)
(a)
γ = a + kb
START
TO (1)
(1)
TO (2)
(2)
FINISH
TOTAL EQUIVILENT COIL


ROTATIONAL AXIS
ϕ_I
ϕ_{II}
i_I
i_{II}


ϕ_I
θ_m
ϕ_0
ϕ_{II}

SINCE ELECTRIC ENERGY IS THE PRODUCT IN SPACE OF THE FLUX OF MAGNETIC INDUCTION, AND THE FLUX OF DIELECTRIC INDUCTION, THE NATURE OF THESE FLUXES, AND THE NATURE OF THEIR PRODUCTS, DETERMINES THE CHARACTERISTICS OF ELECTRIC ENERGY THAT APPEAR IN THE TESLA OSCILLATING CURRENT TRANSFORMER. IT IS THUS IMPORTANT TO INVESTIGATE THE NATURE OF THESE COMPONENTS OF ELECTRIC ENERGY.

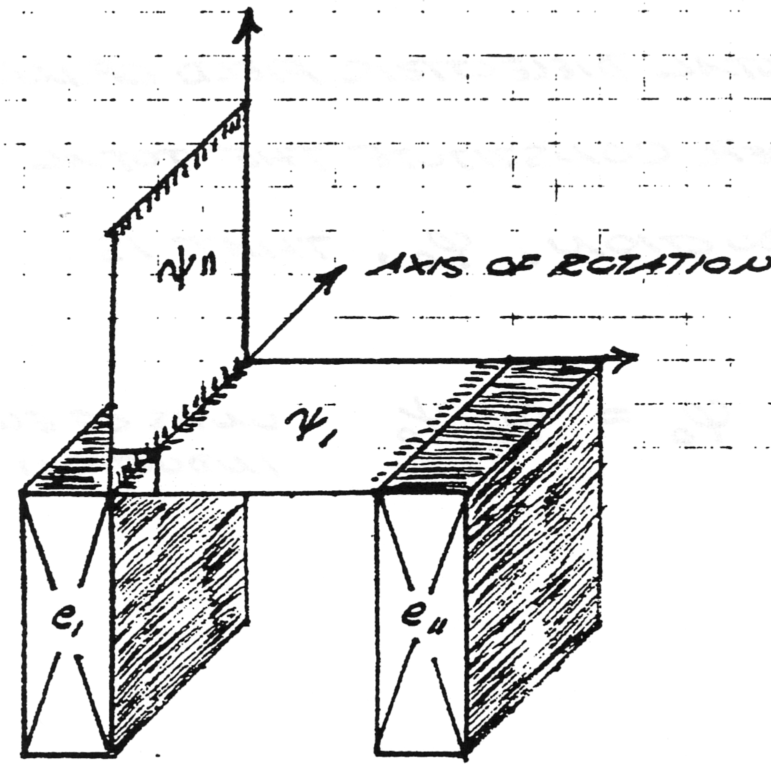
WHEN ELECTRIC ENERGY EXISTS IN ANY SYSTEM OF ELECTRIC "CONDUCTORS" CERTAIN PHENOMENA APPEAR IN THE SPACE SURROUNDING THE CONDUCTORS, THAT IS MAGNETIC AND DIELECTRIC ACTIONS MANIFEST THEMSELVES IN THE SURROUNDING AETHER.

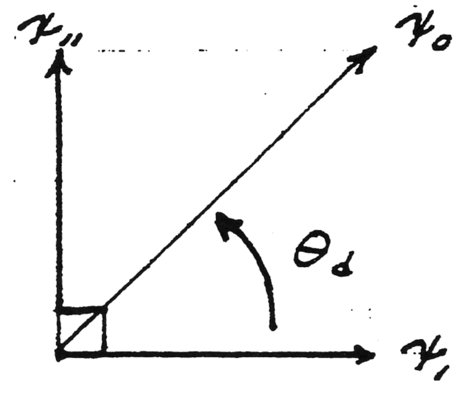
SURROUNDING THE CONDUCTORS IS WHAT IS CALLED THE MAGNETIC FIELD OF INDUCTION. THE INTENSITY OF THIS MAGNETIC FIELD IS GIVEN BY THE TOTAL NUMBER OF MAGNETIC LINES, Φ_0, FILLING THE SURROUNDING SPACE. THE PORTION OF THE TOTAL MAGNETIC INDUCTION WHICH IS PARALLEL TO THE SURFACE OF THE CONDUCTOR IS CALLED

THE TRANSVERSE MAGNETIC INDUCTION, ϕ_{11}, AND THAT PORTION OF THE TOTAL MAGNETIC INDUCTION WHICH IS PERPENDICULAR TO THE SURFACE OF THE CONDUCTORS IS CALLED THE LONGITUDINAL MAGNETIC INDUCTION, ϕ_{1}. IN GENERAL THE TRANSVERSE MAGNETIC INDUCTION EXISTS AT RIGHT ANGLES TO THE FLOW OF ENERGY AND THE LONGITUDINAL MAGNETIC INDUCTION EXISTS IN LINE WITH THE FLOW OF ENERGY. THE GEOMETRIC RELATIONS ARE GIVEN IN FIGURE (2).

ISSUING FROM THE SURFACE OF THE CONDUCTORS IS WHAT IS CALLED THE DIELECTRIC FIELD. THE INTENSITY OF THE DIELECTRIC FIELD IS GIVEN BY THE TOTAL NUMBER OF THE DIELECTRIC LINES OF INDUCTION, Ψ_0. THE PORTION OF THE TOTAL DIELECTRIC INDUCTION THAT TERMINATES UPON SURFACES IN THE DIRECTION OF THE FLOW OF ENERGY IS CALLED THE LONGITUDINAL DIELECTRIC INDUCTION, Ψ_1, AND THE PORTION THAT TERMINATES UPON SURFACES PERPENDICULAR TO THE FLOW OF ENERGY IS CALLED THE TRANSVERSE DIELECTRIC INDUCTION. THE GEOMETRIC RELATIONS ARE GIVEN IN FIGURE (3)

FIG (3)





THE TOTAL MAGNETIC FIELD OF INDUCTION, ϕ_0 AND THE TOTAL DIELECTRIC FIELD OF INDUCTION, Ψ_0, TOGETHER CONSTITUTE THE TOTAL ELECTRIC FIELD OF INDUCTION, φ_0, THAT IS

$$\varphi_0 = \phi_0 \Psi_0$$ UNITS OF ELECTRIC INDUCTION

b) TRANSVERSE AND LONGITUDINAL COMPONENTS

TRANSVERSE ELECTRO-MAGNETIC WAVES, SOMETIMES CALLED HERTZIAN WAVES, ARE THE RESULT OF THE PERPENDICULAR CROSSING IN SPACE OF LINES OF DIELECTRIC INDUCTION, , AND LINES OF MAGNETIC INDUCTION, ϕ, FIGURE (4).

THE SYMBOLIC EXPRESSION OF THIS GEOMETRIC RELATION IS

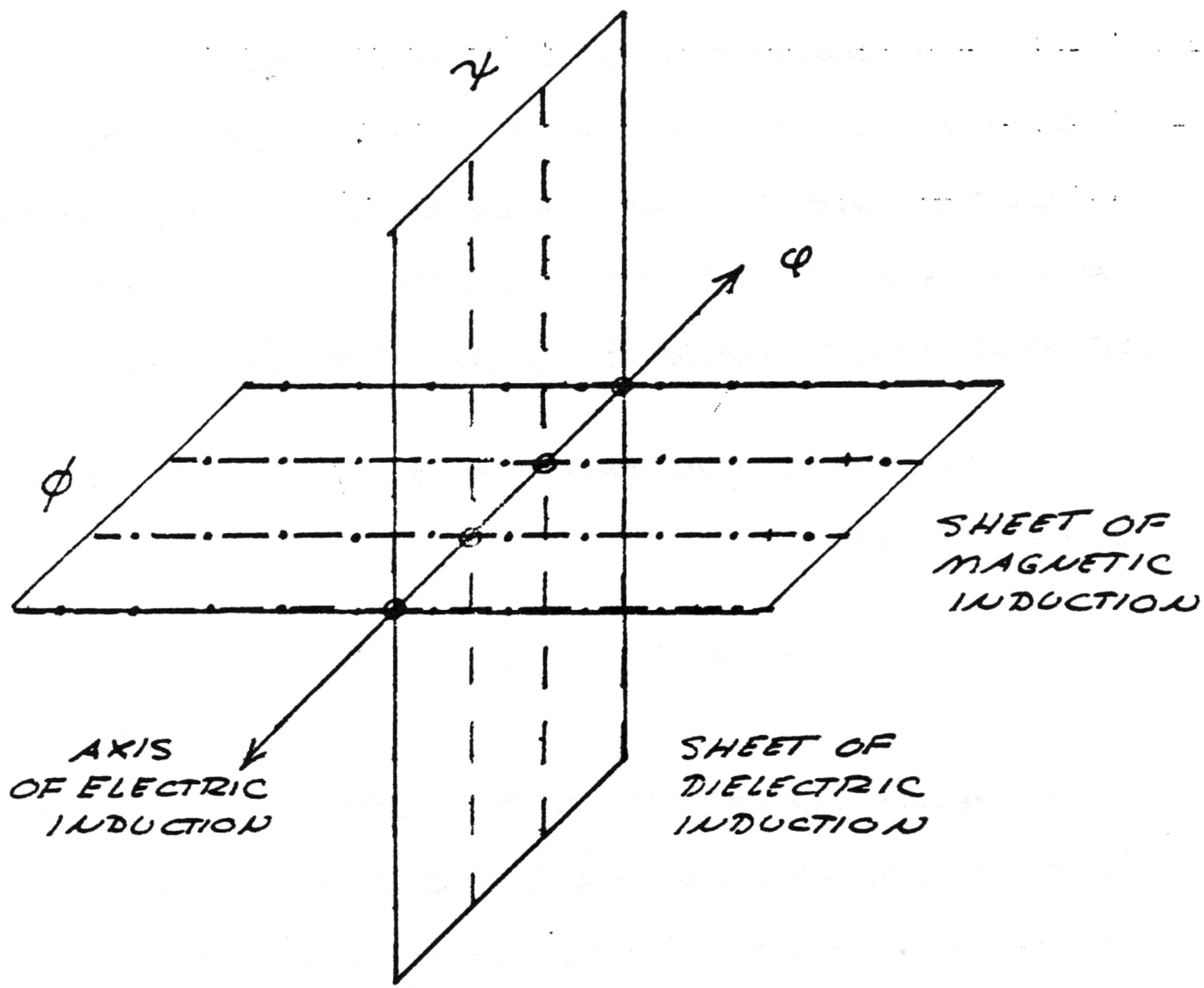
$$\varphi = \psi \times \phi \tag{1}$$

THIS RELATION IS CALLED THE CROSS PRODUCT OF THE MAGNETIC AND DIELECTRIC INDUCTIONS THAT CONSTITUTE THE ELECTRIC INDUCTION. THIS RELATION IS THE BASIS FOR WHAT IS KNOWN AS THE POYNTING VECTOR, FIRST DISCOVERED BY OLIVER HEAVISIDE.

THE TRIGONOMETRIC EXPRESSION OF THIS RELATION IS

$$\varphi = \varphi_0 \sin \theta$$

WHERE θ IS THE ANGLE OF CROSSING BETWEEN THE LINES OF Ψ AND THE LINES OF ϕ.



ψ
φ
ϕ
SHEET OF MAGNETIC INDUCTION
AXIS OF ELECTRIC INDUCTION
SHEET OF DIELECTRIC INDUCTION

SINE TERM

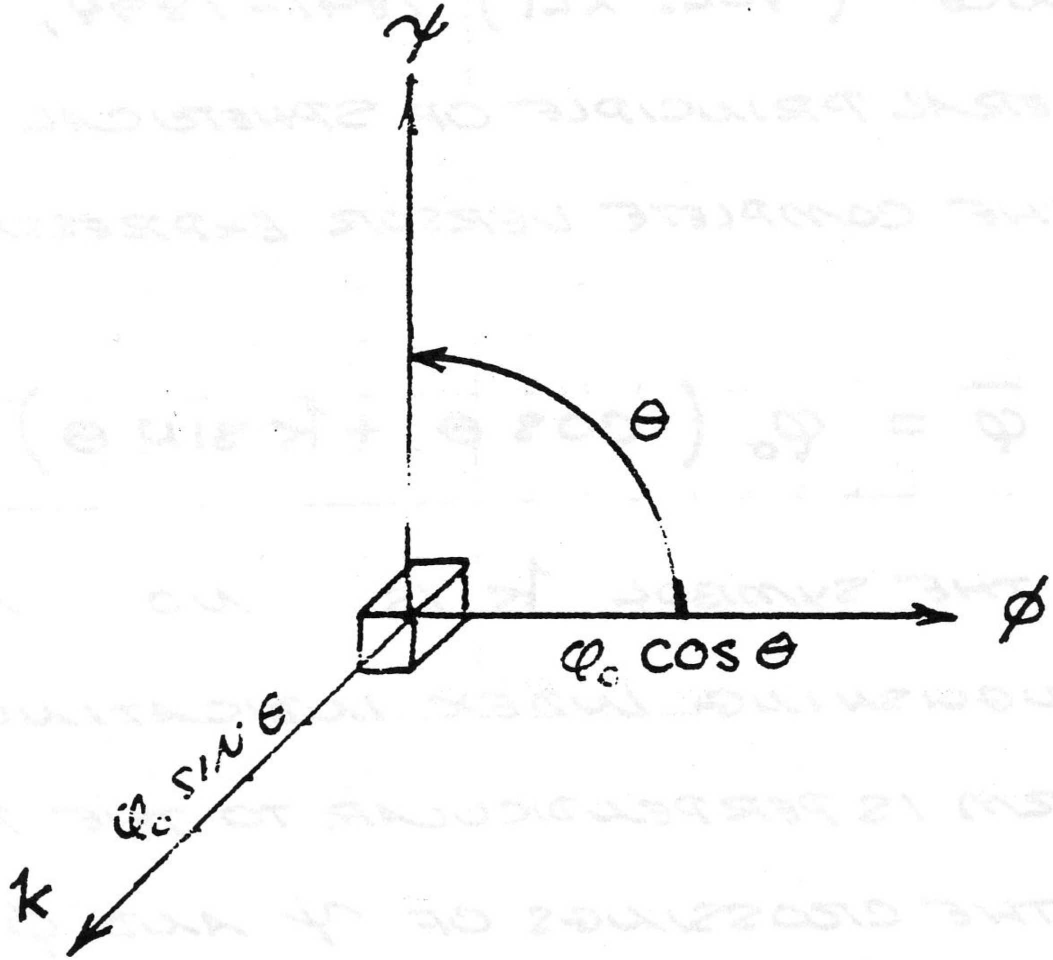
IT WAS SHOWN BY PROF. ALEXANDER MACFARLANE IN THE IMAGINARY OF ALGEBRA PRESENTED BEFORE THE AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE (VOL. XLI) 1891-1894, THAT IT IS A GENERAL PRINCIPLE OF SPHERICAL TRIGONOMETRY THAT THE COMPLETE VERSOR EXPRESSION OF φ IS

$$\bar{\varphi} = \varphi_0 \left(\cos\theta + k \sin\theta \right) \qquad (3)$$

WHERE THE SYMBOL k IS NO MORE THAN A DISTINGUISHING INDEX INDICATING THAT THE SINE TERM IS PERPENDICULAR TO THE PLANE IN WHICH THE CROSSINGS OF ψ AND ϕ OCCUR, FIGURE (5)..

BY SUBSTITUTING THE RELATIONS

$$\left.\begin{aligned} \varphi_{I} &= \varphi_0 \cos\theta \\ \varphi_{II} &= \varphi_0 \sin\theta \end{aligned}\right\} \qquad (4)$$



ψ
θ
ϕ
$\varphi_0 \cos\theta$
$\varphi_0 \sin\theta$
k

THE SYMBOLIC EXPRESSION OF THE COMPLEX INDUCTION IS GIVEN BY

$$\bar{\varphi} = \varphi_{I} + k\varphi_{II} \qquad (5)$$

HENCE, THE FLUX OF ELECTRO-MAGNETIC INDUCTION IS DIRECTED PERPENDICULAR TO THE INDUCTIONS WHICH GIVE RISE TO IT, PROPAGATING IN THE DIRECTION k.

THE DIMENSIONS OF ELECTRO-MAGNETIC ENERGY ARE GIVEN BY

$$W = mc^2 \qquad \text{WATT} \cdot \text{SEC}$$

$$= m\frac{l^2}{t^2} \qquad (6)$$

AND THE DIMENSION OF MAGNETIC FLUX ARE

$$\phi = \frac{i}{W} \qquad \text{LINES} \qquad (7)$$

$$= \frac{i^2}{t}\,\frac{m}{\psi} \qquad (8)$$

SUBSTITUTING EQUATION (7) INTO (6) AND

SUBSTITUTING THE LAW OF DIELECTRIC INDUCTION

$$i = \psi / t \qquad \text{LINES / SEC}$$

GIVES THE DIMENSIONS OF THE TRANSVERSE

ELECTRO-MAGNETIC INDUCTION AS

$$\varphi_{11} = mc^2 T \qquad \text{WATT} \cdot \text{SEC}^2$$

$$= m \frac{l^2}{t} \qquad (9)$$

WHERE T IS THE TIME INTERVAL DURING WHICH ENERGY IS EXCHANGED BETWEEN MAGNETIC AND DIELECTRIC FORMS OF ENERGY STORAGE. THE DIMENSIONS OF EQUATION (9) USUALLY ARE GIVEN AS THE NUMERICAL QUANITY

$$\varphi_{11} = 6.6234 \times 10^{-34} \text{ WATT} \cdot \text{SEC}^2.$$

OR INTEGER MULTIPLES THEREOF. THIS IS USUALLY PORTRAID AS A FLUX OF THESE UNITS OF ENERGY·TIME FLOWING ALONG DIRECTION k, CALLED A FLUX OF PHOTONS.

THE FUNDAMENTAL RELATION GIVEN BY THE EQUATION (3) INDICATES THAT THE ELECTRO-MAGNETIC INDUCTION φ_{11} IS ONLY A PARTIAL COMPONENT OF THE COMPLETE ELECTRIC INDUCTION, $\overline{\varphi}$, DUE TO THE EXISTANCE OF THE COMPLIMENTARY COMPONENT

$$\varphi_1 = \cos\theta$$

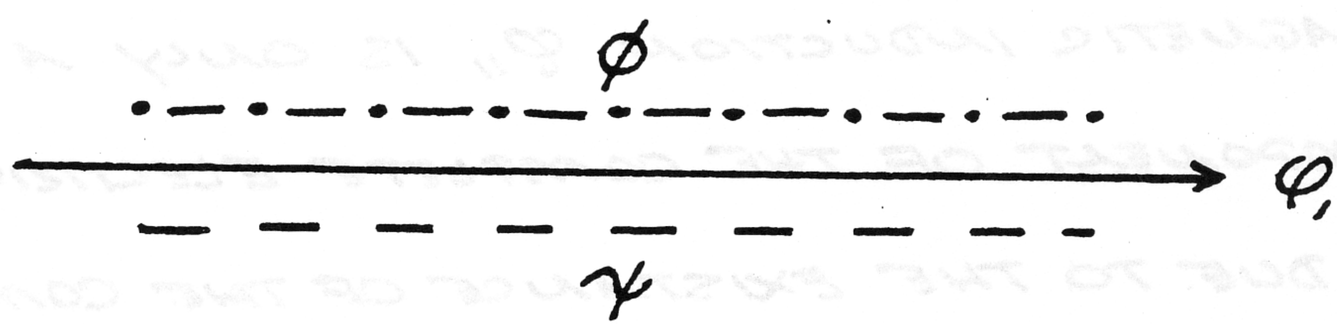
THE GEOMETRIC RELATION OF φ_1 IS SHOWN IN FIGURE (6). THE LINES OF INDUCTION, ψ AND ϕ IN THIS CASE ARE IN SPACE CONJUNCTION AND THUS LAY UPON THE SAME AXIS AS THE FLUX OF ELECTRIC INDUCTION φ_1 TO WHICH THEY GIVE RISE

HENCE, A DISTINCT FORM OF ELECTRIC INDUCTION TOTALLY UNLIKE THE ELECTRO-MAGNETIC COMPONENT φ_{11}. THE SYMBOLIC EXPRESSION OF THIS RELATION IS

$$\varphi_1 = \psi \cdot \phi \qquad (10)$$

FIG (6)

CONJUNCT LINES



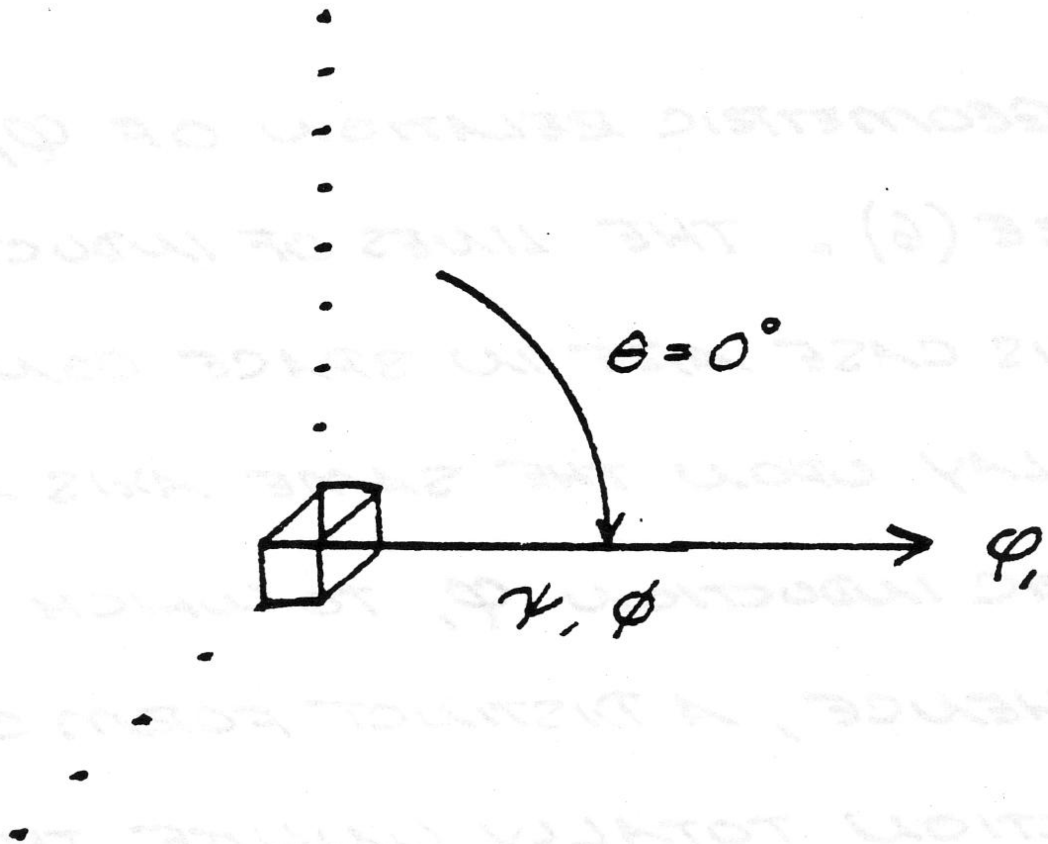
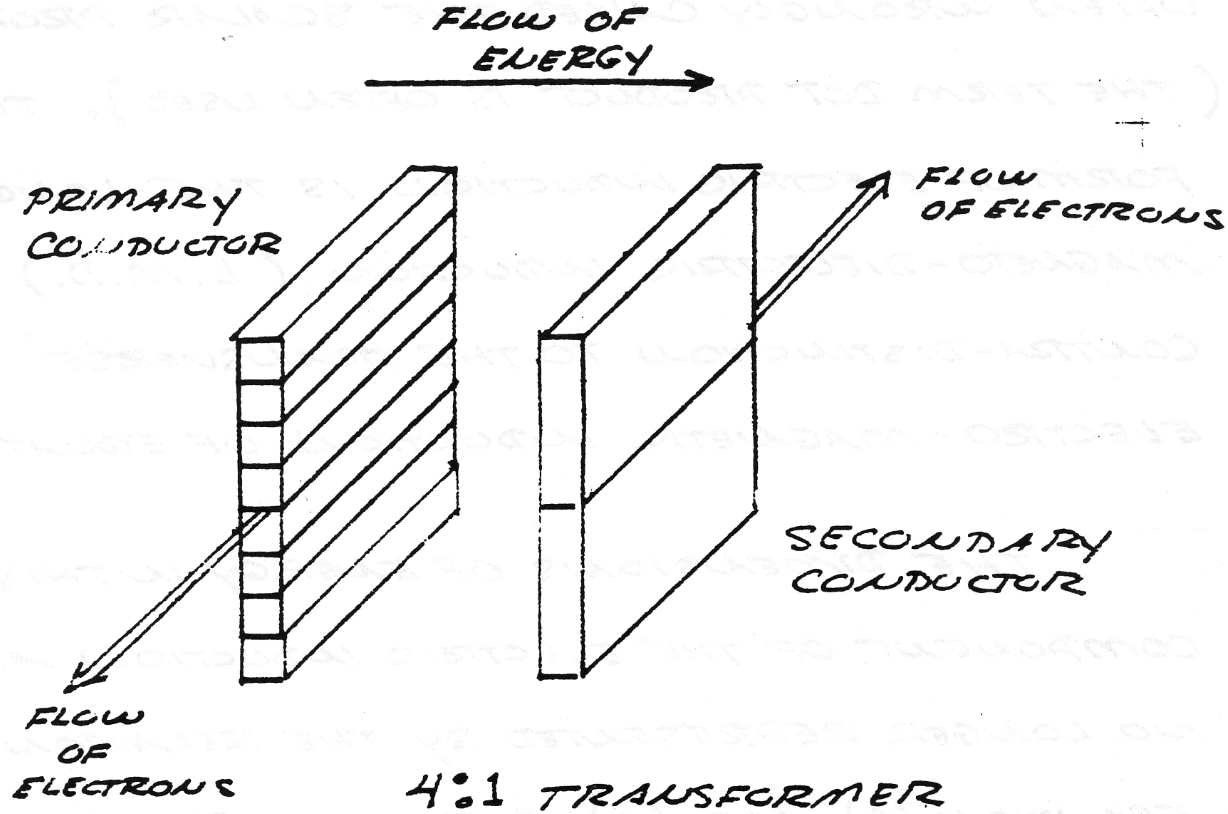


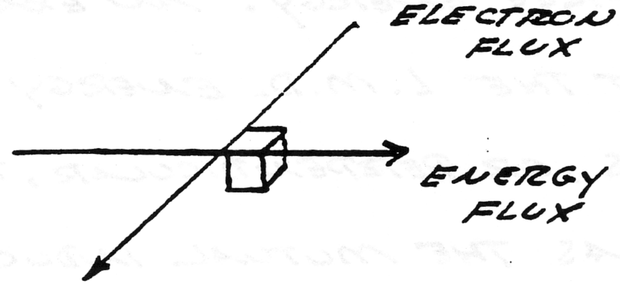
FIG (7)

THIS RELATION IS CALLED THE AXIAL PRODUCT, OFTEN WRONGLY CALLED THE SCALAR PRODUCT (THE TERM DOT PRODUCT IS OFTEN USED). THIS FORM OF ELECTRIC INDUCTION IS THE LONGITUDINAL MAGNETO-DIELECTRIC INDUCTION (L.M.D.) IN CONTRA-DISTINCTION TO THE TRANSVERSE ELECTRO-MAGNETIC INDUCTION OF EQUATION (1).

THE DIMENSIONS OF ENERGY IN THIS COMPONENT OF THE ELECTRIC INDUCTION ARE NO LONGER REPRESENTED BY THE RELATIONS IN EQUATION (6) AND (9) BUT MUST BE REPRESENTED AS A MASS FREE ENERGY. AN EXAMPLE OF THIS FACT IS THAT THE L.M.D. ENERGY PROPAGATES AT RIGHT ANGLES, OR PERPENDICULAR, TO TO ELECTRONIC FLUX, SUCH AS THE MUTUAL INDUCTANCE OF A TRANSFORMER, BEING LONGITUDINAL IN FORM, CONVEYS ENERGY FROM THE PRIMARY COIL TO THE SECONDARY COIL PERPENDICULAR TO THE FLUX OF ELECTRONS IN THE COIL CONDUCTORS, FIGURE (8)



4:1 TRANSFORMER



THE SYMBOLIC EXPRESSION OF THE TOTAL ELECTRIC INDUCTION OF A SYSTEM OF CONDUCTORS IS THE COMPLEX SUM OF THAT PERCENTAGE OF INDUCTION CONTAINED IN THE TRANSVERSE COMPONENT

$$a = \sin\theta \qquad \text{PERCENT} \qquad (11)$$

AND THAT PERCENTAGE OF INDUCTION CONTAINED BY THE LONGITUDINAL COMPONENT

$$b = \cos\theta \qquad \text{PERCENT} \qquad (12)$$

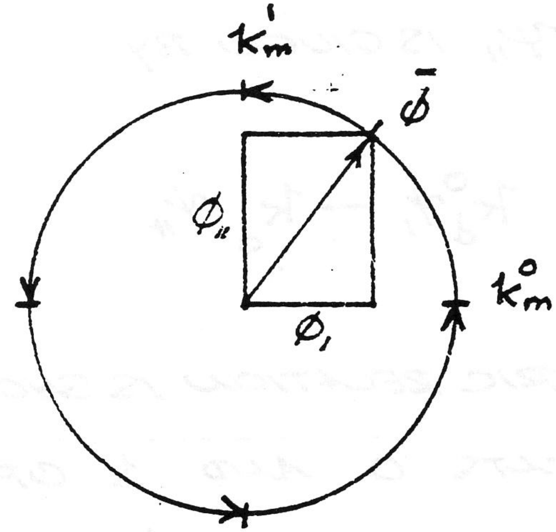
HENCE, THE COMPLEX QUANITY

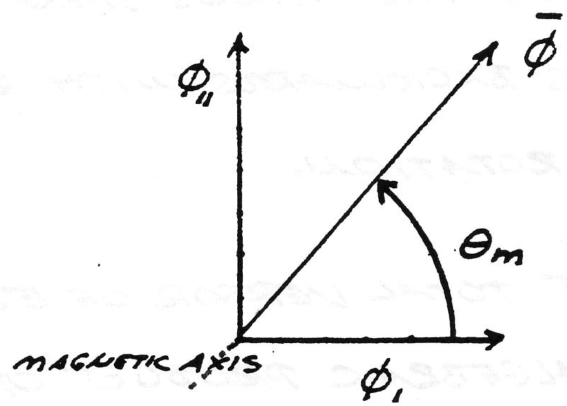
$$\gamma = a + kb \qquad \text{UNITS} \qquad (13)$$

THE VERSOR EQUATION OF ELECTRIC INDUCTION IS HEREBY GIVEN AS

$$\bar{\varphi} = \gamma \varphi_0 \qquad (14)$$

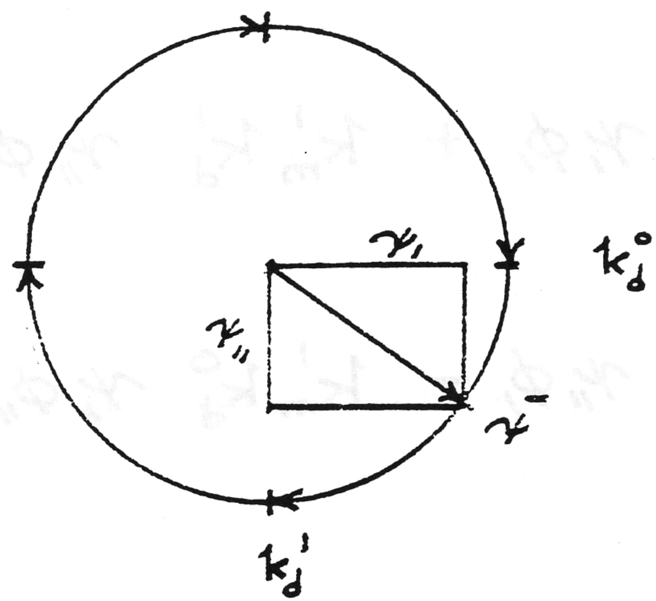
VERSOR DIAGRAM

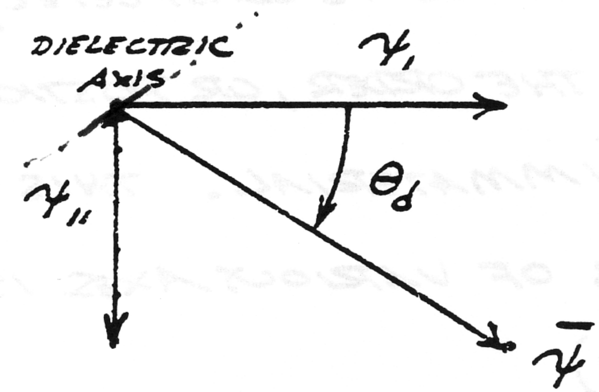




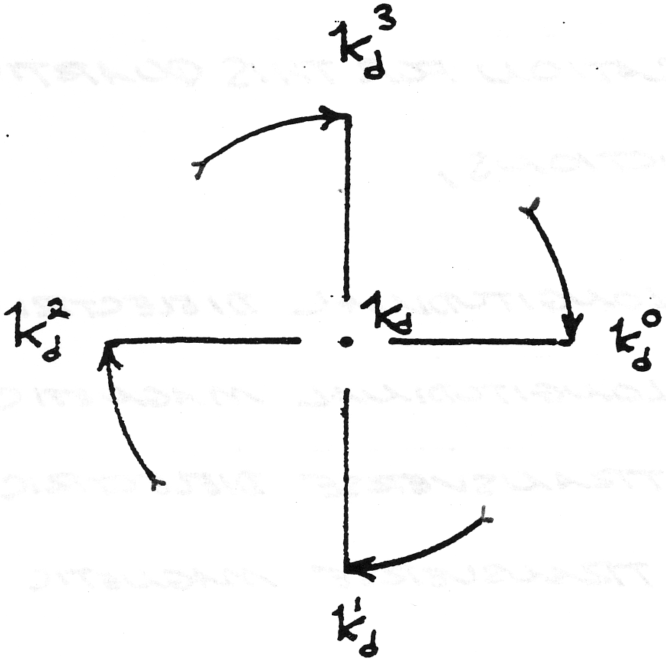
VECTOR DIAGRAM

VERSOR DIAGRAM

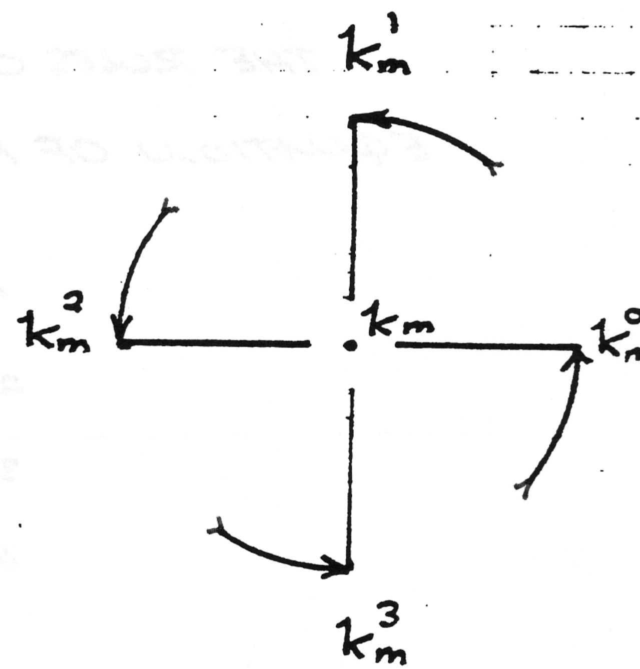




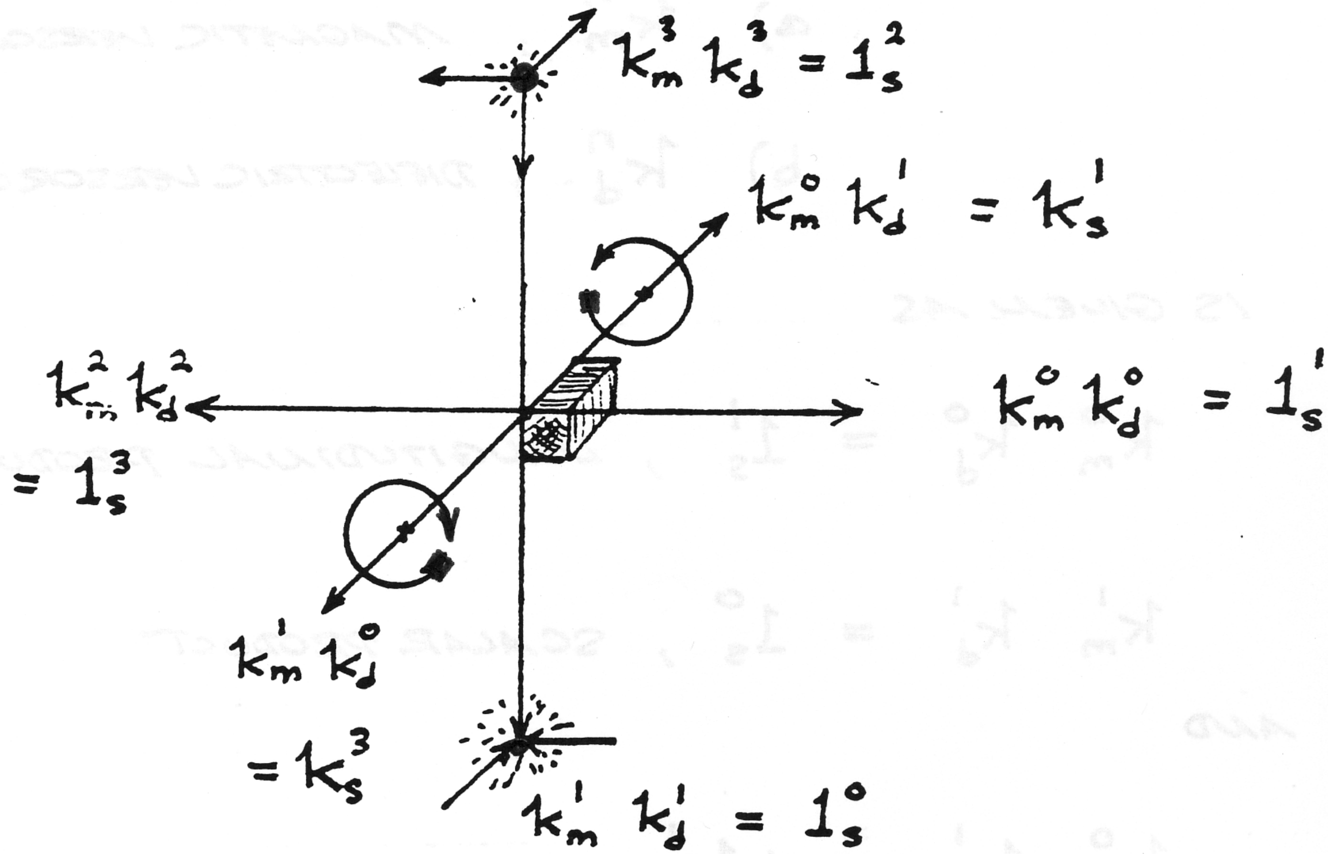
VECTOR DIAGRAM



DIELECTRIC AXIS



MAGNETIC AXIS



QUADRUPLE VERSOR PRODUCT

THE RULES OF MULTIPLICATION FOR THIS QUARTIC EQUATION OF FOUR INDUCTIONS,

1) Ψ_{I} , LONGITUDINAL DIELECTRIC
2) Φ_{I} , LONGITUDINAL MAGNETIC
3) Ψ_{II} , TRANSVERSE DIELECTRIC
4) Φ_{II} , TRANSVERSE MAGNETIC

AND FOR THE CO-AXIAL VERSOR AXES

a) k_m^n , MAGNETIC VERSOR OPERATOR

b) k_d^n , DIELECTRIC VERSOR OPERATOR

IS GIVEN AS

$$k_m^0 \; k_d^0 = 1_s^1 \quad \text{, LONGITUDINAL PRODUCT}$$

$$k_m^1 \; k_d^1 = 1_s^0 \quad \text{, SCALAR PRODUCT}$$

AND

$$k_m^0 \; k_d^1 = +k_s^1 \quad \text{, COUNTER CLOCKWISE CROSS PRODUCT}$$

$$k_m^1 \; k_d^0 = -k_s^1 \quad \text{, CLOCKWISE CROSS PRODUCT}$$

THE SYMBOLS

$1_s^o = 1$ REPRESENTS A DIMENSIONLESS UNIT

$-k_s^1 = k_s^3$ = REPRESENTS TWO QUADRANTS OF ROTATION (180°)

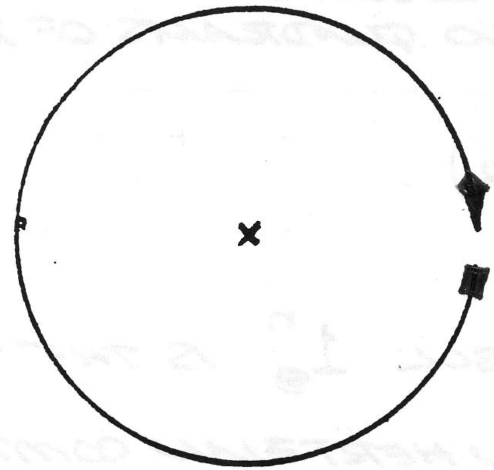
(SEE FIGURE 12)

THE SYMBOL 1_s^n IS THE SPACE OPERATOR FOR THE NON HERTZIAN COMPONENT OF THE VERSOR OF ELECTRIC INDUCTION AND POSSESES THE UNIQUE PROPERTY OF BEHAVING LIKE THE VERSOR, OR TIME OPERATOR, DESCRIBED IN <u>SYMBOLIC REPRESENTATION OF THE GENERALIZED ELECTRIC WAVE</u>, PUBLISHED BY BORDERLAND SCIENCES RESEARCH FOUNDATION, VISTA CA. 92083.

THE SYMBOL k_s^n IS THE SPACE OPERATOR FOR THE HERTZIAN COMPONENT, OR CIRCULARLY POLARIZED T.E.M. COMPONENT, OF THE VERSOR OF ELECTRIC INDUCTION.

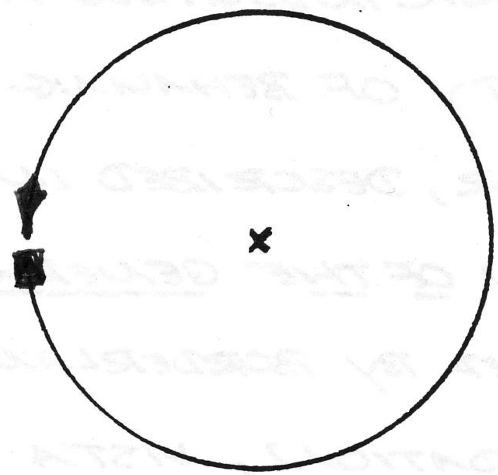
C.W CROSS PRODUCT

$1k_s^{-1}$



C.C.W CROSS PRODUCT

$1k_s^{+1}$



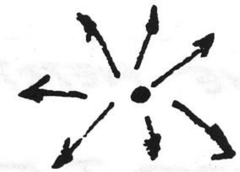
FWD. LINE PRODUCT

1_s^{+1}

REV. LINE PRODUCT

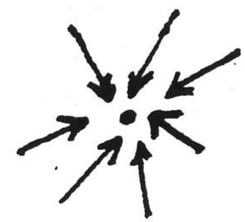
1_s^{-1}

-1_s°



EXPANDING DOT PRODUCT

$+1_s^{\circ}$



CONTRACTING DOT PRODUCT

SUBSTITUTING THE VALUES OF THE MULTIPLICATION RULES INTO EQUATION (18) GIVES THE SYMBOLIC EXPRESSION

$$\bar{\varphi} = (\phi_{11}\psi_{11} + 1_s^1 \phi_1\psi_1)$$

$$+ k_s^1 (\phi_1\psi_{11} - \phi_{11}\psi_1) \qquad (19)$$

SUBSTITUTING

$$\bar{\varphi}_1 = (\phi_{11}\psi_{11} + 1_s^1 \phi_1\psi_1) \qquad (20A)$$

AND

$$\bar{\varphi}_{11} = (\phi_1\psi_{11} - \phi_{11}\psi_1) \qquad (20B)$$

GIVES THE GENERAL EXPRESSION OF THE VERSOR OF COMPLEX INDUCTION IN A FORM SIMILAR TO EQUATION (5),

$$\bar{\varphi} = \bar{\varphi}_1 + k_s^1 \bar{\varphi}_{11} \qquad (21)$$

$\phi_I \Psi_{II}$; REPRESENTS THAT COMPONENT OF THE ELECTRIC INDUCTION THAT IS TRANSVERSE ELECTRO-MAGNETIC IN FORM AND IS VERTICALLY POLARIZED. THIS INDUCTION EXHIBITS VARIATION TRANSVERSE, OR PERPENDICULAR, TO THE TRANSFORMER WINDING'S AXIS, AND PASSES THRU THE SPACE BETWEEN THE CONDUCTOR TURNS IN A COUNTER CLOCKWISE DIRECTION.

$\phi_{II} \Psi_I$; REPRESENTS THAT COMPONENT OF THE ELECTRIC INDUCTION THAT IS TRANSVERSE ELECTRO-MAGNETIC IN FORM AND IS HORIZONTALLY POLARIZED. THIS INDUCTION EXHIBITS VARIATION TRANSVERSE, OR PERPENDICULAR, TO THE TRANSFORMER WINDING'S AXIS, AND PASSES THRU THE SPACE AROUND THE OUTSIDE OF THE WINDING IN A CLOCKWISE DIRECTION.

IT CAN BE SEEN THAT THE TWO TRANSVERSE INDUCTIONS REPRESENT A PAIR OF TRAVELLING WAVES MOVING IN OPPOSITE DIRECTIONS AROUND THE WINDING.

WHILE THE SCALAR INDUCTION FILLS ALL SPACE SURROUNDING THE TRANSFORMER AND DOES NOT PROPAGATE IT DOES PULSATE IN TIME AND THEREFORE IS NOT SCALAR IN THE DIMENSION OF TIME, BUT REPRESENTS THE TRUE L.C. OSCILLATION OF THE TRANSFORMER AS A LUMPED CIRCUIT.

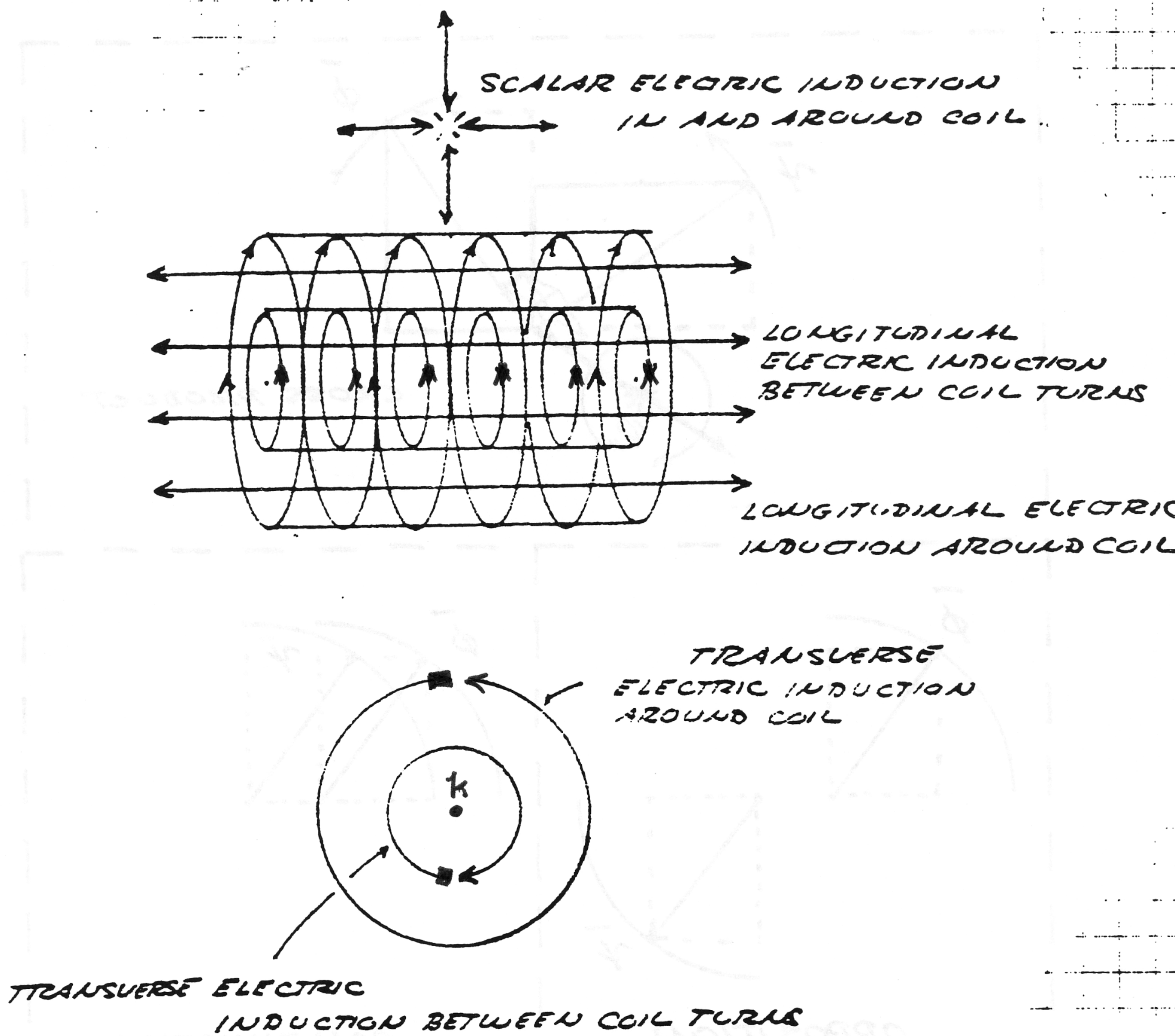
$$LC = T^2$$

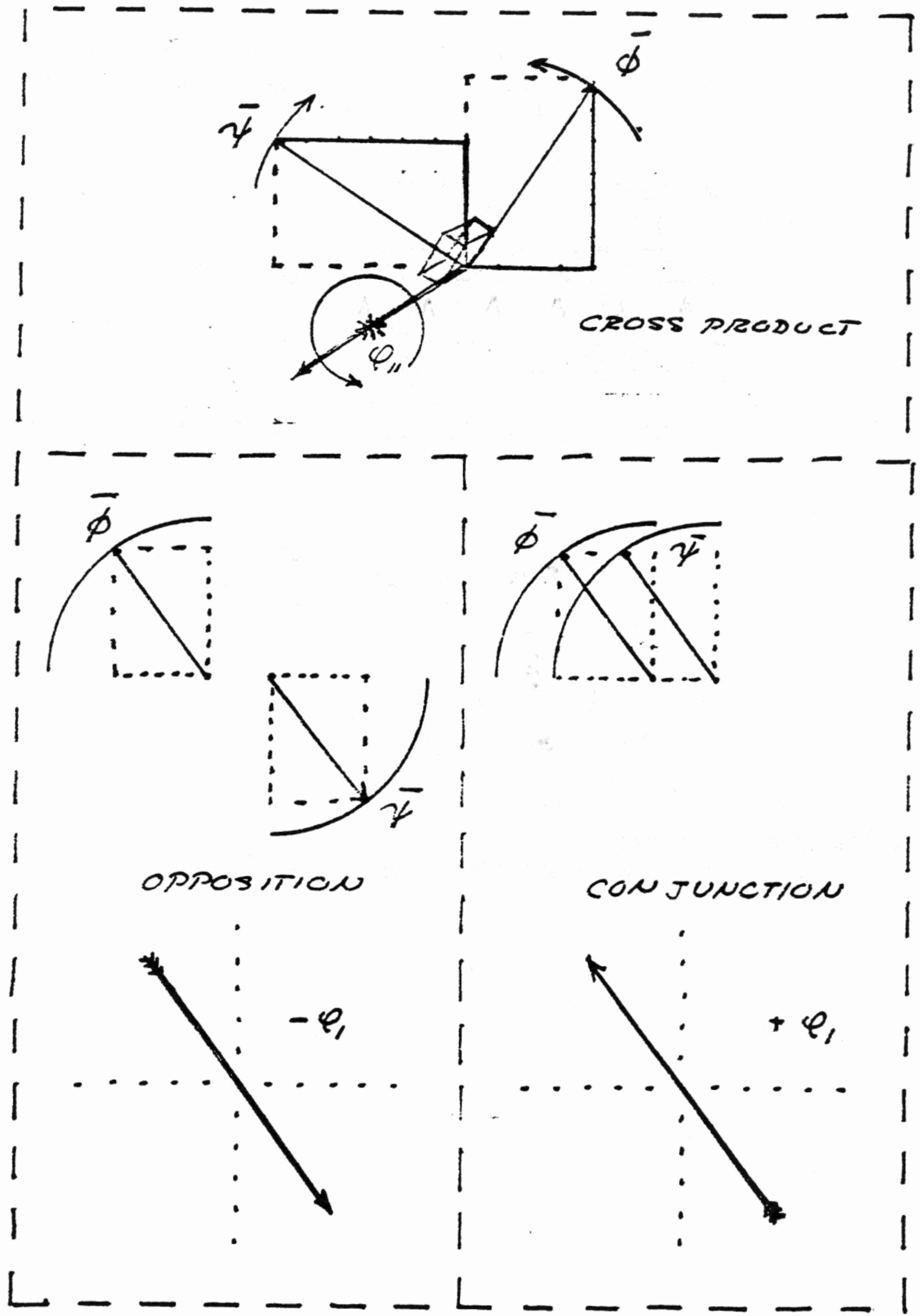
HENCE, THE COMPLETE TRANSFORMER OSCILLATION WITH SPATIAL VARIATION IS GIVEN BY

$$(LC - u_s^2) = T_o^2 \qquad (22)$$

WHERE u_s IS CALLED THE SPACE CONSTANT.

THE VARIOUS INDUCTIONS AND THERE RELATION TO THE TRANSFORMER WINDING ARE SHOWN BY FIGURE (13)


SCALAR ELECTRIC INDUCTION
IN AND AROUND COIL.
LONGITUDINAL
ELECTRIC INDUCTION
BETWEEN COIL TURNS
LONGITUDINAL ELECTRIC
INDUCTION AROUND COIL
TRANSVERSE
ELECTRIC INDUCTION
AROUND COIL
k
TRANSVERSE ELECTRIC
INDUCTION BETWEEN COIL TURNS



$\bar{\psi}$
$\bar{\phi}$
φ_{11}
CROSS PRODUCT
$\bar{\phi}$
$\bar{\psi}$
OPPOSITION
$-\varphi_1$
$\bar{\phi}$
$\bar{\psi}$
CONJUNCTION
$+\varphi_1$

THE TRANSVERSE COMPONENT OF THE ELECTRIC INDUCTION IS NEUTRALIZED IF THE CONDITION EXISTS THAT

$$\overline{\varphi}_{11} = \left(\phi_1 \psi_1 - \phi_{11} \psi_{11} \right) = \text{ZERO} \qquad (23)$$

AND THEREFORE

$$\frac{\psi_{11}}{\psi_1} = \frac{\phi_{11}}{\phi_1}$$

HENCE

$$\text{TAN}\ \theta_d = \text{TAN}\ \theta_m$$

THIS IS SHOWN BY FIGURE (14).

ALTERNATLY, EQUATION (23) BECOMES

$$\frac{\phi_1}{\psi_1} = \frac{\phi_{11}}{\psi_{11}}$$

THAT IS

$$Z_1 = Z_{11}$$

THE CHARACTERISTIC IMPEDANCE OF THE LONGIT-UDINAL ELECTRIC INDUCTION IS EQUAL TO THE CHARACTERISTIC IMPEDANCE OF THE SCALAR INDUCTION.

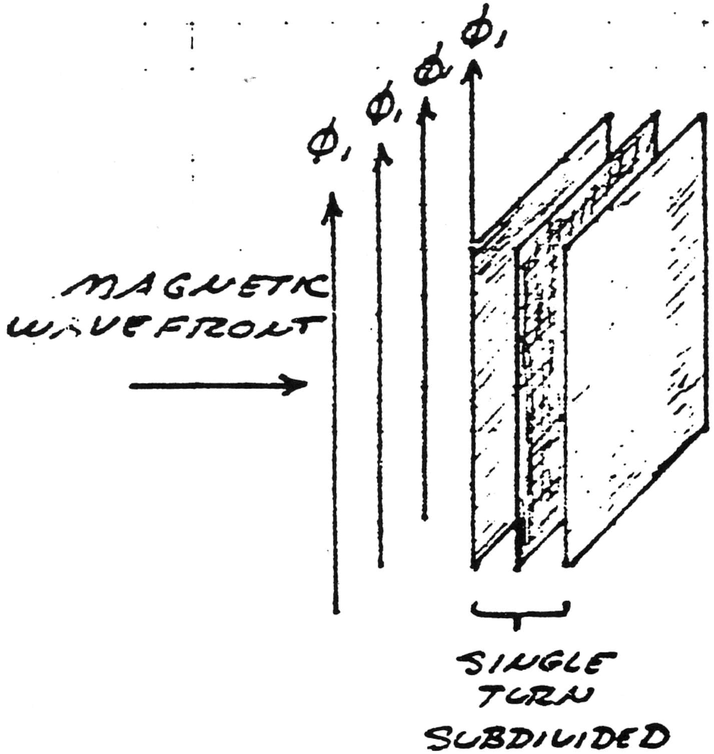
IN THIS CASE OF NEUTRALIZED T.E.M. THE WINDING MAY BE SAID TO BE DISTORTIONLESS, THEREBY PRODUCING AN UNDISTORTED HARMONIC WAVEFORM IN OSCILLATION.

THE NON-HERTZIAN COMPONENT OF THE ELECTRIC INDUCTION IS NEUTRALIZED IF THE CONDITION EXISTS THAT

$$\bar{\varphi}_1 = \left(\phi_1 \Psi_1 \;\; + 1_s' \, \phi_{11} \Psi_{11} \right) = \text{ZERO}$$

WHICH DOES NOT SEEM POSSIBLE SINCE THE TWO TERMS MUST BE COMPLEX QUALITIES.

IT SHOULD BE NOTED THAT THE PRESENCE OF THE CONDUCTOR MATERIAL SERVES TO DISTORT THE DISTRIBUTION OF INDUCTION BECAUSE IT EXCLUDES THE MAGNETIC INDUCTION BY THE PRODUCTION OF EDDY CURRENTS. FOR THIS REASON THE CONDUCTOR MATERIAL MUST BE LAMINATED IN A FASHON SIMILAR TO THAT FOUND IN TRANSFORMER CORES, FIGURE (15). LITZ WIRE WILL SERVE AS LAMINATION IN THE WINDING OF O.C. TRANSFORMERS.



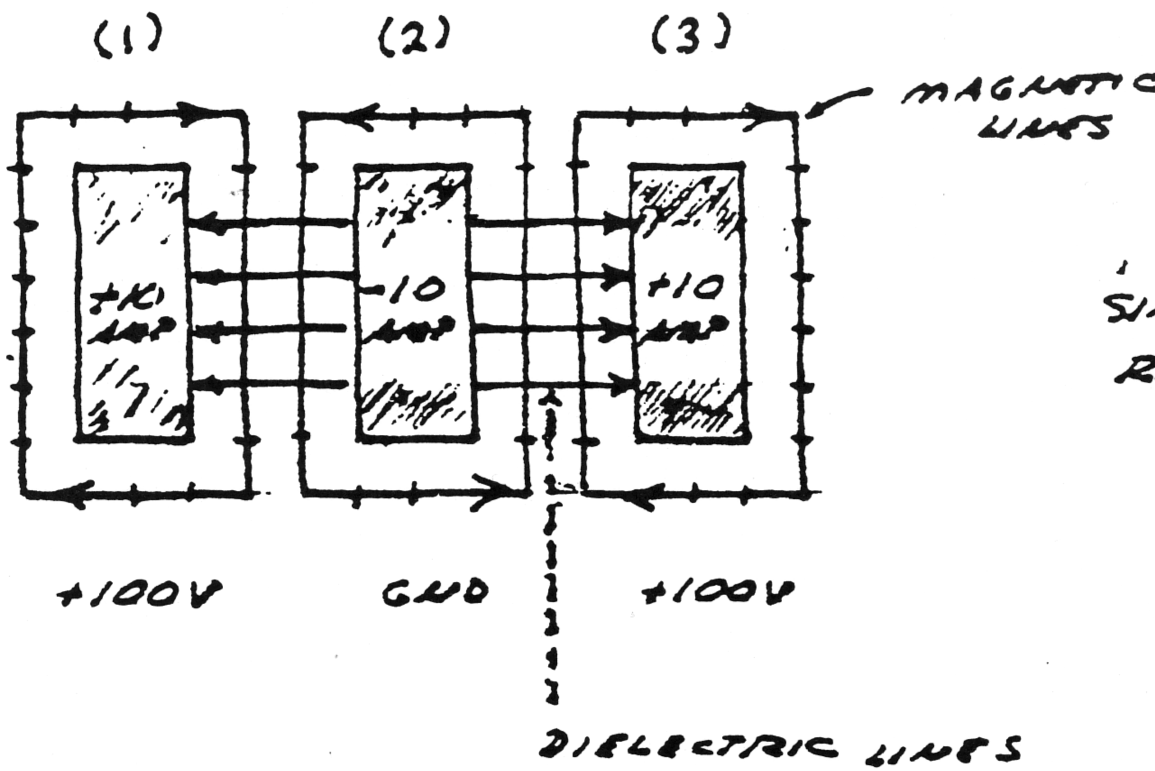
THICKNESS OF CONDUCTOR SHEETS MUST BE LESS THAN 10% OF THE SKIN DEPTH OF ELECTRONIC CONDUCTION AT OPERATING FREQUENCY, OR MAXIMUM HARMONIC THEREOF ($\approx 15F_0$). FOR 1000 K_c SEC. THIS IS LESS THAN 0.001 INCH.

The Oscillating Current Transformer

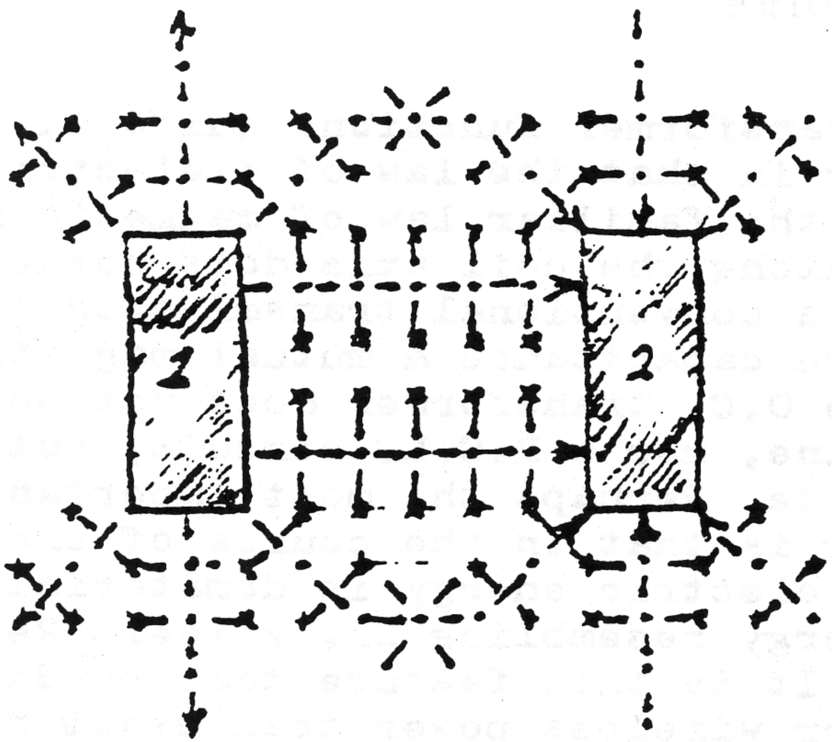
The oscillating current transformer functions quite differently than a conventional transformer in that the law of dielectric induction is utilized as well as the familiar law of magnetic induction. The propagation of waves along the coil axis does not resemble the propagation of waves along a conventional transmission line, but is complicated by inter-turn capacitance & mutual magnetic inductance. In this respect the O.C. transformer does not behave like a resonant transmission line, nor a R.C.L. circuit, but more like a special type of wave guide. Perhaps the most important feature of the O.C. transformer is that in the course of propagation along the coil axis the electric energy is dematerialized, that is, rendered mass free energy resembling Dr. Wilhelm Reich's Orgone Energy in its behavior. It is this feature that renders the O.C. transformer usefull for wireless power transmission and reception, and gives the O.C. transformer singular importance in the study of Dr. Tesla's research.

FUNDAMENTALS OF COIL INDUCTION

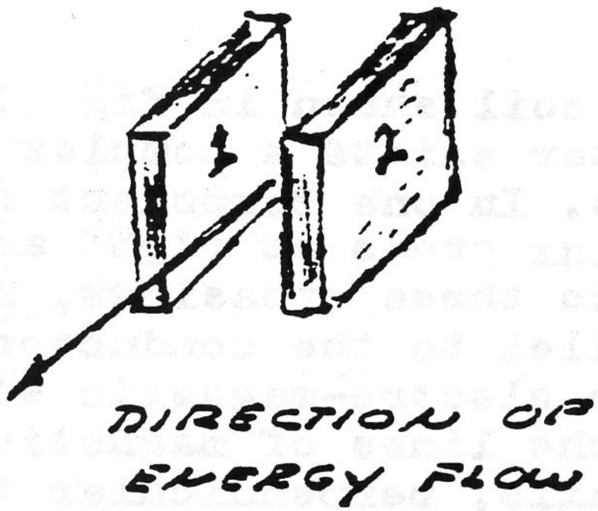
Consider the elemental slice of a coil shown in fig. 1. Between the turns 1,2 & 3 of the coiled conductor exists a complex electric wave consisting of two basic components. In one component (fig. 2), the lines of magnetic and dielectric flux cross at right angles, producing a photon flux perpendicular to these crossings, hereby propagating energy along the gap, parallel to the conductors and around the coil. This is the transverse electro-magnetic wave. In the other component, shown in fig. 3, the lines of magnetic flux do not cross but unite along the same axis, perpendicular to the coil conductors, hereby energy is conveyed along the coil axis. This is the Longitudinal Magneto-Dielectric Wave.



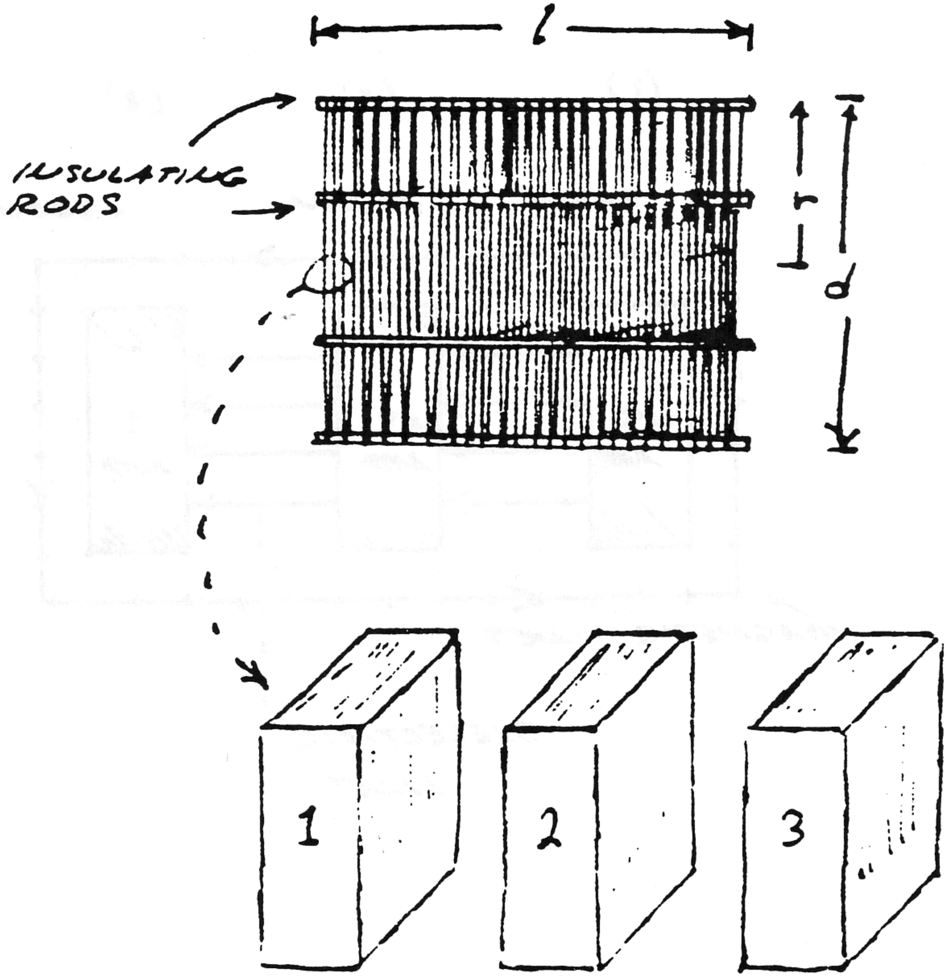
SIMPLE FLUX RELATIONS



COMPOSITE FLUX PATTERN OF TWO TURNS



TRANSVERSE WAVE



SOLENOIDAL COIL OF N TURNS AND WIRE LENGTH OF l_0.

SLICE OF COIL WINDING, THREE TURNS

Hence, two distinct forms of energy flow are present in the coiled conductor, propagating at right angles with respect to each other, as shown in fig. 4. Hereby a resultant wave is produced which propagates around the coil in a helical fashion, leading the transverse wave between the conductors. Thus the oscillating coil posses a complex wavelength which is shorter than the wavelength of the coiled conductor.

COIL CALCULATION

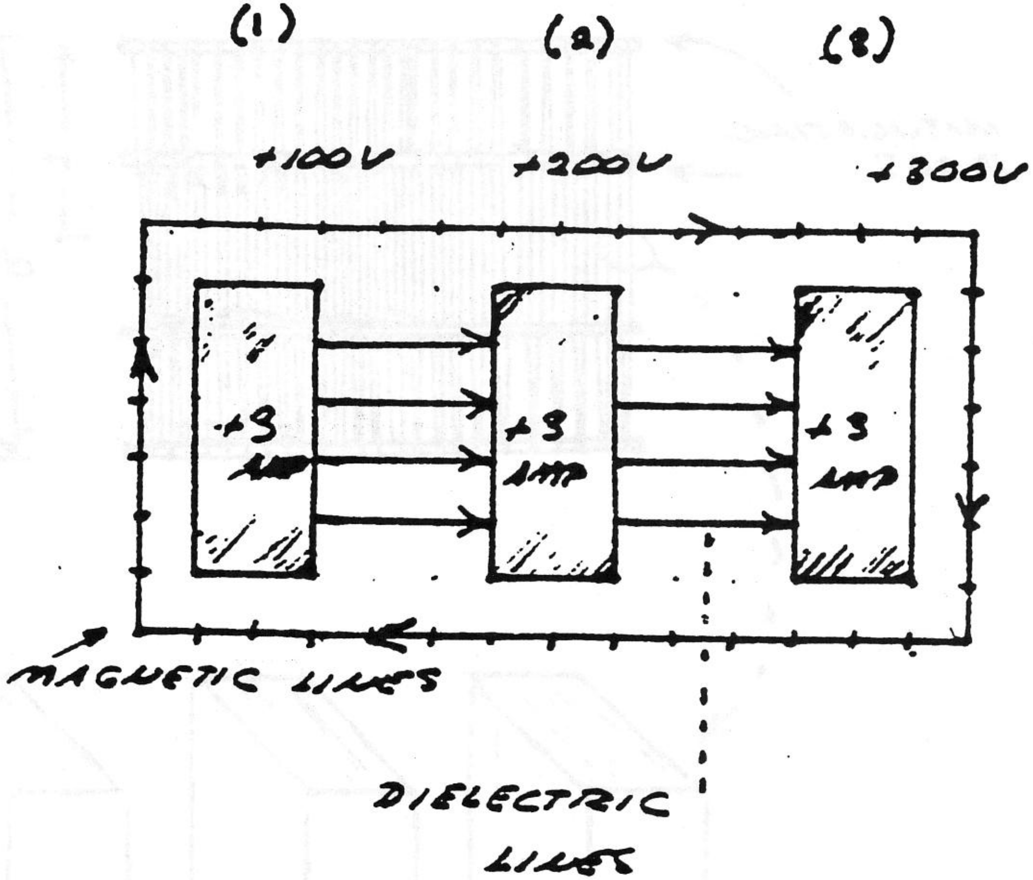
If the assumptions are made that an alternating current is applied to one end of the coil, the other end of the coil is open circuited. Additionaly external inductance and capacitance must be taken into account, then simple formulae may be derived for a single layer solenoid.

The well known formula for the total inductance of a single layer solenoid is

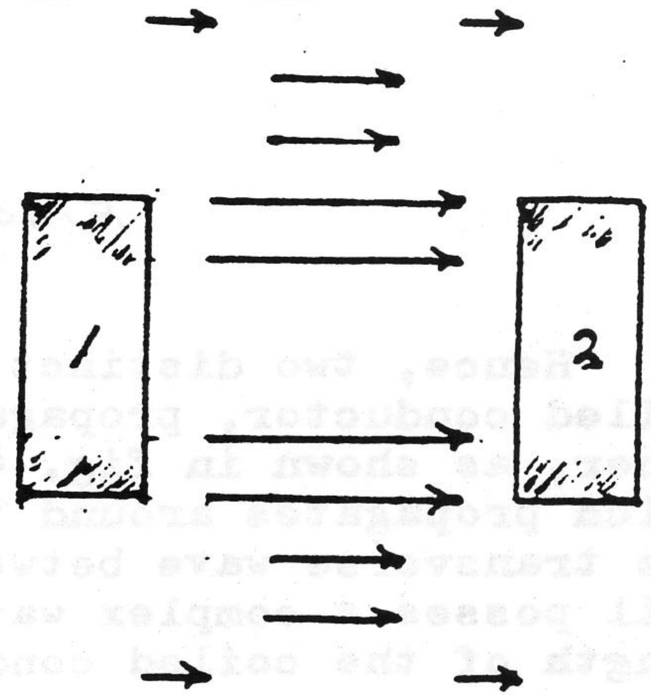
$$L = \frac{r^2 N^2}{(9r+101)} \times 10^{-6} \text{ Henry (inches)} \qquad (1)$$

Where
- r is coil radius
- l is coil length
- N is number of turns

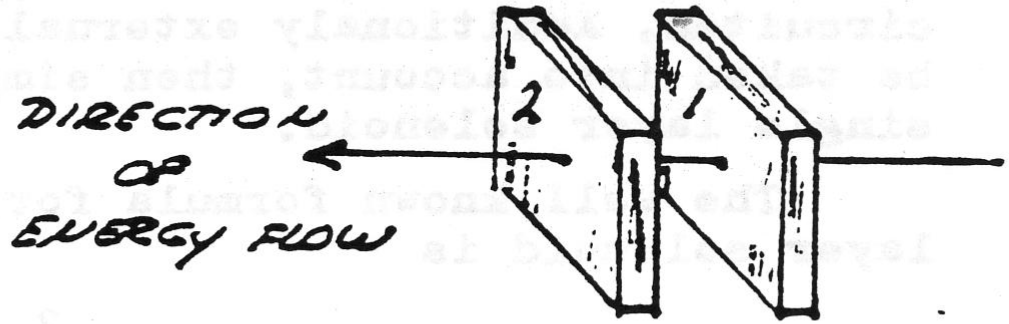
SIMPLE FLUX RELATIONS



COMPOSITE FLUX PATTERN OF TWO TURNS



LONGITUDINAL WAVE



The capacitance of a single layer solenoid is given by the formula

$$C = pr \qquad 2.54 \times 10^{-12} \text{ Farads (inches)} \qquad (2)$$

where the factor p is a function of the length to diameter ratio, tabulated in table (1). The dimensions of the coil are shown in figure (1). The capacitance is minimum when length to diameter ratio is equal to one.

Because the coil is assumed to be in oscillation with a standing wave, the current distribution along the coil is not uniform, but varies sinusoidially with respect to distance along the coil. This alters the results obtained by equation (1), thus for resonance

$$L_o = \tfrac{1}{2}L \qquad \text{Henrys} \qquad (3)$$

likewise, for capacitance

$$C_o = \frac{8}{\pi} C \qquad \text{Farads} \qquad (4)$$

Hereby the velocity of propagation is given by

$$V_o = 1/\sqrt{L_o C_o} \qquad \text{Units/sec} \qquad (5)$$

$$= \eta V_c$$

Where

$$V_c = 1/\sqrt{\mu \varepsilon} \qquad \text{Inch/sec} \qquad (6)$$

That is, the velocity of light, and

$$V_o = V_c \eta$$

$$= \left[\frac{1.77}{p} + \frac{3.94}{p} n \right]^{\frac{1}{2}} \quad 2\pi 10^9 \text{ Inch/sec} \qquad (7)$$

Where

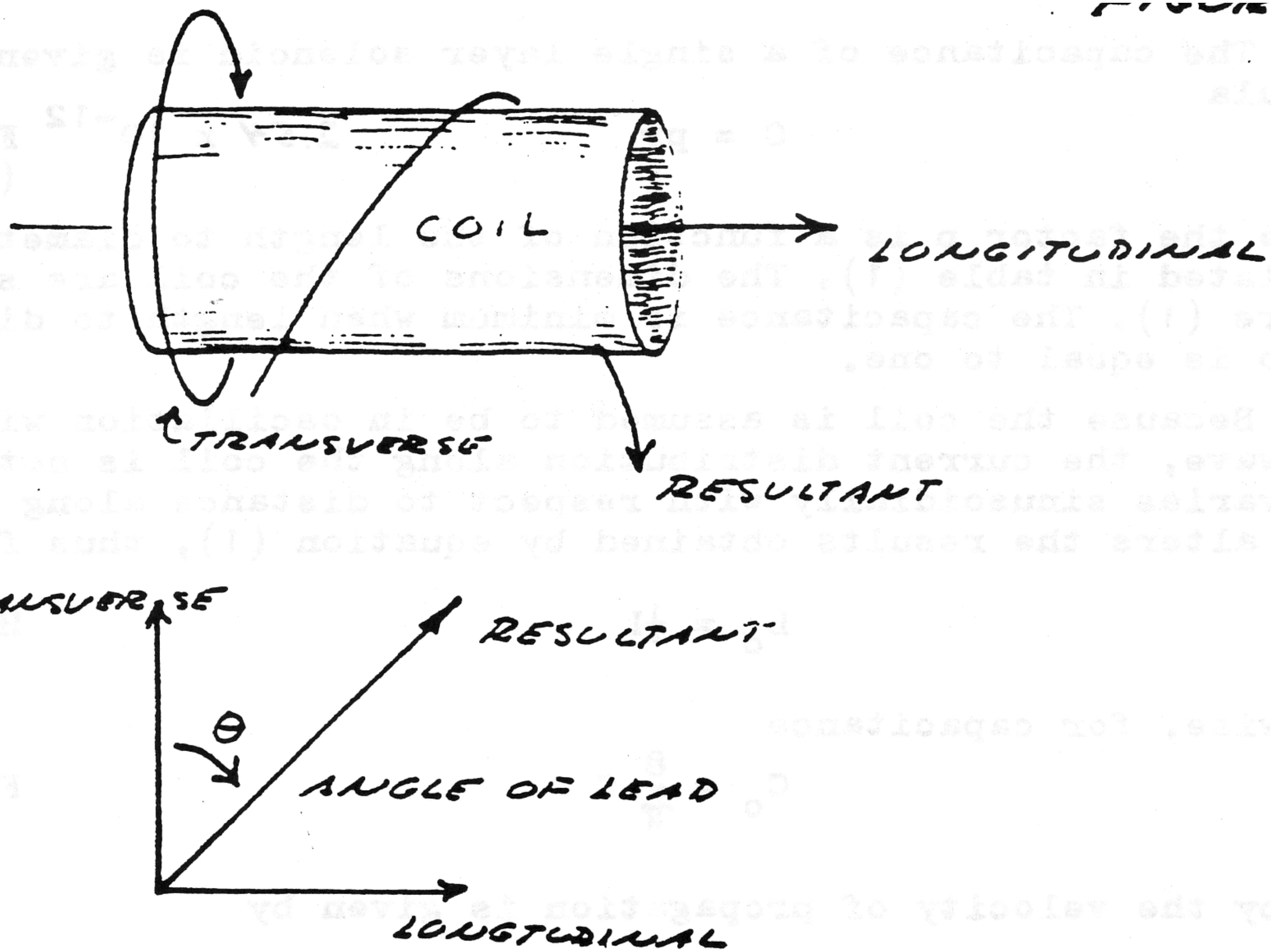
n = the ratio of coil length to coil diameter. The values of propagation factor η are tabulated in table (2).

Thus, the frequency of oscillation or resonance of the coil is given by the relation

$$F_o = V_o/(l_o \cdot 4) \qquad \text{Cycles/sec} \qquad (8)$$

Where

l_o = total length of the coiled conductor in inches.



COIL
LONGITUDINAL
TRANSVERSE
RESULTANT
TRANSVERSE
RESULTANT
θ
ANGLE OF LEAD
LONGTUDINAL

The characteristic impedance of the resonant coil is given by

$$Z_c = \sqrt{\frac{L_o}{C_o}} \qquad \text{Ohms} \qquad (9)$$

Hence,

$$Z_c = NZ_s \qquad \text{Ohms} \qquad (10)$$

Where

$$Z_s = \left[(182.9 + 406.4n)p\right]^{\frac{1}{2}} \qquad \frac{\pi}{2}\,10^3 \qquad \text{Ohms (inches)} \qquad (11)$$

and

N = number of turns. The values of sheet impedance, Z_s, are tabulated in table (3).

The time constant of the coil, that is, the rate of energy dissipation due to coil resistance is given by the approximate formula

$$u = R_o/2L_o = \left(\frac{2.72}{r} + \frac{2.13}{l}\right)\pi\sqrt{F_o} \qquad \text{Nepers/sec (inches)} \qquad (12)$$

Where

r = coil radius

l = coil length

In general, the dissipation of the coil's oscillating energy by conductor resistance:

1) Decreases with increase of coil diameter, d;

2) Decreases with increase of coil length, l, rapidly when the ratio, n, of length to diameter is small with little decrease beyond n equal to unity;

3) Is minimum when the ratio of wire diameter to coil pitch is 60%.

By examination of the attached tables, (1), (2) & (3), it is seen that the long coils of popular designs do not result in optimum performance. In general, coils should be short and wide, and not longer than n=1. The frequency is usually given as $F_o = V_c/\lambda_o$ which by equation (7) is incorrect. Winding on solid or continous formers rather than spaced slender rods, as shown in figure (1), greatly retards wave propagation as indicated in equation (6), thereby seriously distorting the wave. The dielectric constant of the coil, ε, should be as close to unity as is physically possible to insure high efficiency of transformation.

The equations for the voltampere relations of the oscillating coil are

$$\dot{E}_1 = j\,(Z_c\,Y_o + \delta\,)\,\dot{E}_o \qquad \text{Complex Input Voltage} \qquad (13)$$

$$\dot{I}_1 = j\,(Y_c\,Z_o + \delta\,)\,\dot{I}_o \qquad \text{Complex Input Current} \qquad (14)$$

$$Z_1 = \frac{Z_c\,Y_o + \delta}{Y_c\,Z_o + \delta}\,Z_o \qquad \text{Input Impedance, Ohms} \qquad (15)$$

Where

$\dot{E}_o$ = Voltage on elevated terminal

$\dot{I}_o$ = Current into elevated terminal

$Y_c = Z_c^{-1}$

Z_o = Terminal impedance

Y_o = Terminal admittance

$\delta = u/2F_o$ = Decrement

j = root of $\sqrt{-1}$

For negligible losses and absolute values

$$E_1 = (Z_c 2\pi F_o C_o) E_o \qquad \text{Volts} \qquad (16)$$

$$I_1 = (Y_c / 2\pi F_o C_o) I_o \qquad \text{Amperes} \qquad (17)$$

Where

C_o = Terminal capacitance

By the law of conservation of energy

$$E_1 I_1 = E_o I_o \qquad \text{Volt-Amperes} \qquad (18)$$

If the terminal capacitance is small then the approximate input/output relations of the Tesla coil are given by

$$E_o = Z_c I_1 \qquad \text{Output Volts} \qquad (19)$$

$$I_1 = E_o Y_c \qquad \text{Input Amperes} \qquad (20)$$

$$I_o = Y_c E_1 \qquad \text{Output Amperes} \qquad (21)$$

$$E_1 = I_o Z_c \qquad \text{Input Volts} \qquad (22)$$

Coil Capacitance Factor

Length/Width = n	Factor p	Length/Width = n	Factor p
0.10	0.96	0.80	0.46
0.15	0.79	0.90	0.46
0.20	0.70	1.00	0.46
0.25	0.64	1.5	0.47
0.30	0.60	2.0	0.50
0.35	0.57	2.5	0.56
0.40	0.54	3.0	0.61
0.45	0.52	3.5	0.67
0.50	0.50	4.0	0.72
0.60	0.48	4.5	0.77
0.70	0.47	5.0	0.81

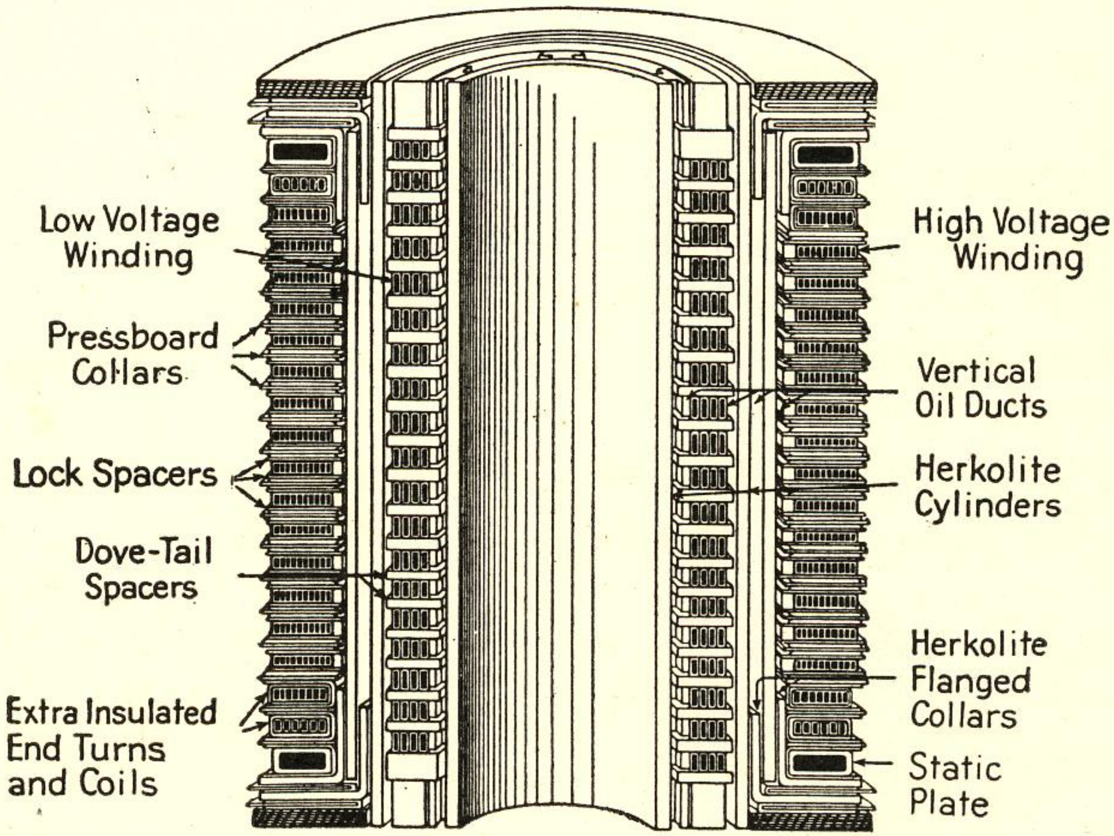


FIG. 97.—Cross-Section of Comparatively High-Voltage Power Transformer

TRANSIENT OSCILLATIONS IN THE PRIMARY WINDINGS *

Numerical calculations covering practical cases, based on the analysis given in the previous chapter, show that the essential characteristics of the oscillations in the primary winding are substantially the same as obtain when the secondary winding is ignored, provided that the Fourier expansion is on the same base in both cases. It is appropriate, therefore, to give the analysis for a single independent winding, because the equations then become greatly simplified and easy to visualize, and a number of characteristic curves can be prepared. It must be borne in mind, however, that it may be necessary to make arbitrary changes in the circuit constants, particularly of the inductance coefficient, to obtain accurate numerical agreement. Also, the minor frequency set disappears in the single-winding theory, and there is no explicit indication of the effect of the secondary in fixing the appropriate Fourier expansion.

The following analysis is along the same lines as that given for the two-winding theory, and is idealized to the same extent, but the effect of the losses and of the applied wave shape is taken into account, and the influence of each of the several circuit constants is discussed. As before, the derivations are restricted to either a grounded or isolated neutral.

THE GENERAL DIFFERENTIAL EQUATION

Referring to Fig. 103*A*, the circuit constants per unit length of winding are:

L = inductance coefficient, including the partial interlinkages.
$M(x, y)$ = mutual inductance between elements at x and y.
C = shunt capacitance to ground.
K = series capacitance along the winding.
G = shunt conductance to ground.

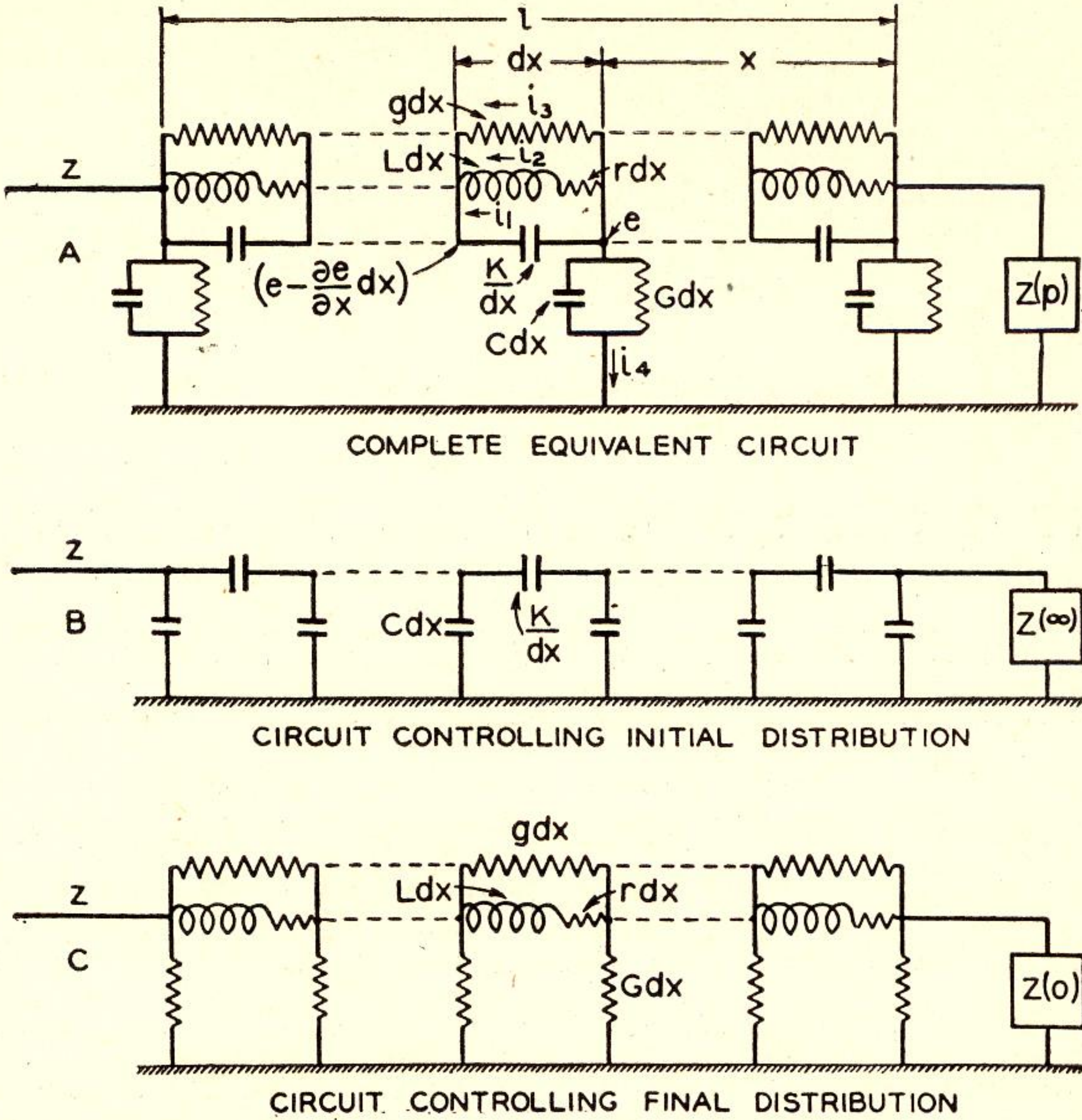


FIG. 103.—Ideal Complete Circuit of a Winding

g = shunt inductance along the winding.
r = series resistance.
n = turns.

The variables involved at any point of the winding are:

e = potential to ground.
i_1 = current in series capacitance K.
i_2 = current in the inductance L.
i_3 = current in the shunt conductance g.
i_4 = current to ground through G and C.
ϕ = total flux linkages at a point.
B = flux density.
t = time.
$p = \partial/\partial t$ = partial derivative with respect to time.
x, y = points along the winding, measured from the neutral end.
l = length of the winding.
$(m\,l\,t)$ = mean length of turn.
$2\,h$ = length of the leakage path.

The fundamental relationships are:

$$i_1 = K \frac{\partial^2 e}{\partial x \, \partial t} \qquad (1)$$

$$i_3 = g \frac{\partial e}{\partial x} \tag{2}$$

$$i_4 = \left(G + C \frac{\partial}{\partial t}\right) e = \frac{\partial}{\partial x} (i_1 + i_2 + i_3) \tag{3}$$

$$\frac{\partial e}{\partial x} = r\, i_2 + \frac{n}{10^8} \frac{\partial \phi}{\partial t} \tag{4a}$$

$$= r\, i_2 + \frac{\partial}{\partial t} \int_0^l M\,(x, y)\, i_2\,(y) \cdot d\,y \tag{4b}$$

$$= r\, i_2 + \frac{\partial}{\partial t} \Big\{ L'\, i_2\,(x)$$

$$+ \int_0^l M\,(x, y)\,[i_2\,(y) - i_2\,(x)]\, d\,y \Big\} \tag{4c}$$

where

$$L' = \int_0^l M\,(x, y)\, d\,y = \text{self inductance}$$

and as in (2) of Chapter XII

$$\phi = \phi_m + \phi_l = \phi_m + \frac{0.4\,\pi\,(m\,l\,t)\,n}{h}\int_0^x \int_x^l i_2\,dy\,dz \tag{5}$$

where

ϕ_m = flux mutual to the entire winding.

ϕ_l = flux due to partial interlinkages.

From (4a) and (5)

$$\frac{\partial^4 e}{\partial x^4} = r\,\frac{\partial^3 i_2}{\partial x^3} - \frac{0.4\,\pi\,n^2\,(m\,l\,t)}{h\,10^8}\,\frac{\partial^2 i_2}{\partial x\,\partial t}$$

$$= r\,\frac{\partial^3 i_2}{\partial x^3} - \frac{L}{l^3}\,\frac{\partial^2 i_2}{\partial x\,\partial t} \tag{6}$$

where

$$L = \frac{0.4\,\pi\,n^2\,l^3\,(m\,l\,t)}{h\,10^8} = \text{effective inductance}$$

By (1), (2), and (3) there is

$$\frac{L}{l^3}\,\frac{\partial^2 i_2}{\partial x\,\partial t} = \frac{L}{l^3}\left(G + C\,\frac{\partial}{\partial t}\right)\frac{\partial e}{\partial t} - \frac{L}{l^3}\,\frac{\partial^2 i_1}{\partial x\,\partial t} - \frac{L}{l^3}\,\frac{\partial^2 i_3}{\partial x\,\partial t}$$

$$= \frac{L}{l^3}\,G\,\frac{\partial e}{\partial t} + \frac{L}{l^3}\,C\,\frac{\partial^2 e}{\partial t^2} - \frac{L}{l^3}\,K\,\frac{\partial^4 e}{\partial x^2\,\partial t^2} - g\,\frac{L}{l^3}\,\frac{\partial^3 e}{\partial x^2\,\partial t} \tag{7}$$

and

$$r \frac{\partial^3 i_2}{\partial x^3} = r\,G \frac{\partial^2 e}{\partial x^2} + r\,C \frac{\partial^3 e}{\partial x^2\,\partial t} - r\,K \frac{\partial^5 e}{\partial x^4\,\partial t} - g\,r \frac{\partial^4 e}{\partial x^4} \tag{8}$$

Substituting (7) and (8) in (6), there results

$$r\,K \frac{\partial^5 e}{\partial x^4\,\partial t} + (1 + g\,r) \frac{\partial^4 e}{\partial x^4} - \frac{L}{l^3} K \frac{\partial^4 e}{\partial x^2\,\partial t^2}$$

$$- \left(r\,C + g \frac{L}{l^3}\right) \frac{\partial^3 e}{\partial x^2\,\partial t} - r\,G \frac{\partial^2 e}{\partial x^2} + \frac{L}{l^3} C \frac{\partial^2 e}{\partial t^2} + \frac{L}{l^3} G \frac{\partial e}{\partial t} = 0 \tag{9}$$

If the losses can be neglected, Equation (9) reduces to

$$\frac{\partial^4 e}{\partial x^4} - \frac{L\,K}{l^3} \frac{\partial^4 e}{\partial x^2\,\partial t^2} + \frac{L}{l^3} C \frac{\partial^2 e}{\partial t^2} = 0 \tag{10}$$

Hereafter it will be convenient to take $l = 1$.

The total current is, from (3)

$$(i_1 + i_2 + i_3) = \left(G + C \frac{\partial}{\partial t}\right) \int e\,d\,x \tag{11}$$

The solutions to these equations must satisfy

a. The differential equation.

b. The terminal conditions at $x = 0$ and $x = l$.

c. The initial distribution at $t = 0$.

d. The final distribution at $t = \infty$.

If the solution corresponding to a constant sustained potential suddenly applied at $x = l$ can be found, then the solution for any other applied terminal voltage is given by Duhamel's theorem. The usual procedure in solving a partial differential equation is to assume the form of the solution and try it by direct substitution in the differential equation and the boundary conditions. Each tentative trial usually suggests the necessary changes and adjustments in order to meet the complete specifications. Therefore, in order to choose the proper solution from among the infinite number of functions which will satisfy the differential equations, it is necessary to first investigate the boundary conditions.

APPENDIX I

EXPERIMENTAL ANALOG COMPUTING NETWORKS FOR THE STUDY OF COMPLEX ELECTRIC WAVES:

1) TRANSVERSE ELECTRO-MAGNETIC PROPAGATION ON SECTION OF T.E.M. TRANSMISSION LINE

FIG (1) LOW PASS FILTER TYPE CHARACTERISTICS, PHASE LAG CONDITION, CAUSE BEFORE EFFECT

FIG (3) HARMONIC RESONANT SERIES, ODD ORDER QUARTER WAVE MULTIPLES, FOWARD SLOPE

2) LONGITUDINAL MAGNETO-DIELECTRIC PROPAGATION ON SECTION OF L.M.D. TRANSMISSION LINE

FIG (1) HIGH PASS FILTER TYPE CHARACTERISTICS, PHASE LEAD CONDITION, EFFECT BEFORE CAUSE

FIG (2) HARMONIC RESONANT SERIES, ODD ORDER QUARTER WAVE MULTIPLES, CONTRARY SLOPE

3) COMPLEX PROPAGATION ON SECTION OF SHUNT CONCA TENATED TRANSMISSION LINE. OSCILLATION TRANSFORMER ANALOG (TESLA TRANSFORMER)

FIG (4) HIGH PASS FILTER TYPE CHARACTERISTICS, PHASE LEAD CONDITION, EFFECT BEFORE CAUSE

FIG (5) EN-HARMONIC RESONANT SERIES, ODD ORDER QUARTER WAVE MULTIPLES, SLOPING CONTRARY SLOPE

FIG (6) TRANSMISSION LINE RESPONSE TO DISRUPTIVE DISCHARGE, CONTINUOUSLY LAGGING PHASE LAG, OR DOWNWARD FREQUENCY SHIFT.

4) COMPLEX PROPAGATION ON SECTION OF SERIES CONCA TENATED TRANSMISSION LINE, ALEXANDERSON AERIAL ANALOG.

FIG (7 TRANSMISSION LINE RESPONSE TO DISRUPTIVE DISCHARGE, DISCONTINUOUSLY LAGGING PHASE LAG, OR DISCONTINUOUS DOWNWARD FREQUENCY SHIFT

FIG (8) BAND PASS FILTER TYPE CHARACTERISTIC, PHASE LAG-LEAD CONDITION, INTERCHANGE OF CAUSE AND EFFECT WITH EFFECT AND CAUSE. SIDE BAND MODULATION CHARACTERISTIC & SCALAR PASSBAND.

FIG (9) COMPOUND EN-HARMONIC SERIES, ODD ORDER QUARTER WAVE MULTIPLES, FOWARD & CONTRARY SLOPES WITH FREQUENCY & AMPLITUDE MODULATION SIDEBANDS.

5) EQUIVELENT CIRCUIT OF COMPENSATED TRANSVER - TRANSMISSION, ALEXANDERSON TYPE

FIG (10) DOUBLE SIDEBAND / DUAL SLOPE ODD ORDER QUARTER WAVE RESONANT.

6) OBSERVATIONS, SIMPLE LINES

a) TRANSVERSE STRUCTURES EXHIBIT HARMONIC MULTIPLES IN RESONANT LINES (WIRES)

b) LONGITUDINAL STRUCTURES EXHIBT HARMONIC DIVISIONS IN RESONANT LINES (STACKS)

c) TRANSVERSE DIMENSIONS, ~~FOWER~~ FOWARD TIME, SPATIAL PROPAGATION

d) LONGITUDINAL DIMENSIONS, REVERSE TIME COUNTER, SPATIAL PROPAGATION

7) OBSERVATIONS, CONCATENATED (COMPLEX) LINES

a) SHUNT (TESLA TRANSFORMER) HARMONICS SIMILAR TO LONGITUDINAL LINE & SAME FOR HIGH PASS CHARACTER

b) TRANSIENT FLOWS SMOOTHLY, STARTS WITH INSTANTANEOUS D.C. LEVEL. CLEAN DOWNWARD SWEEP OF OSCILLITORY FREQUENCY

c) SERIES (ALEXANDERSON AERIAL) HARMONICS IN TWO GROUPS, ONE SIMILAR TO TRANSVERSE AND A SECOND SIMILAR TO LONGITUDINAL. NO D.C. START, SLOW OSCILLATORY BUILDUP. FACTORS FIGHTING EACH OTHER AND TRANSIENT ENDS AS NOISE.

d) DISTINCT FLAT PASS BAND REGION. STRONG MODULATOR LIKE CHARACTERISTICS INDICATING NON-LINEAR TRANSFORMATION.

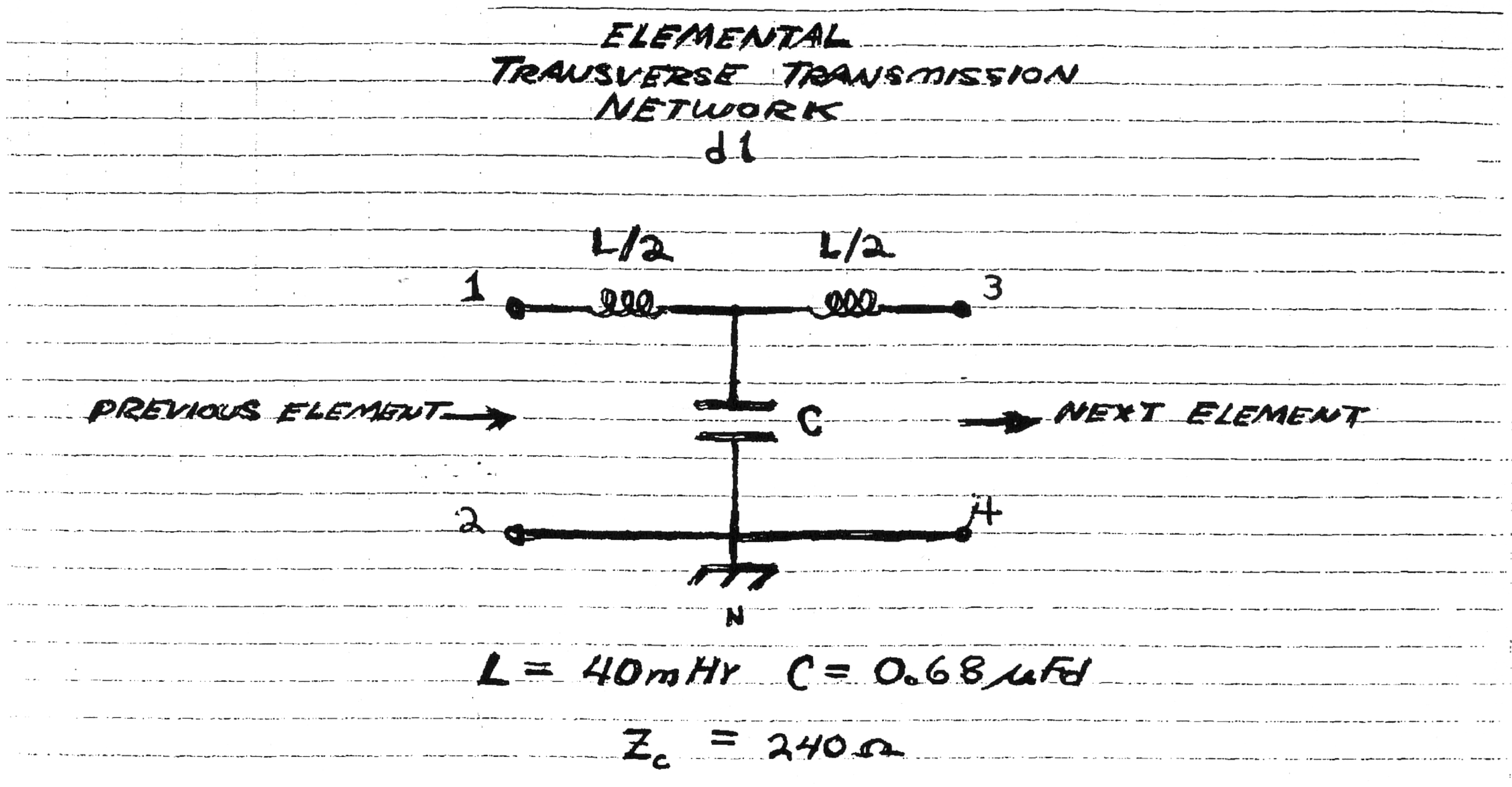
e) SERIES CONCATENATED NETWORK APPEARS TO REQUIRE MORE CRITICAL BALANCING IN ORDER TO BE HARMONIUS. SHUNT CONCACTENATED NETWORK A MORE NATURAL, OR HARMONIUS RESPONSE.

8) CONCLUSIONS

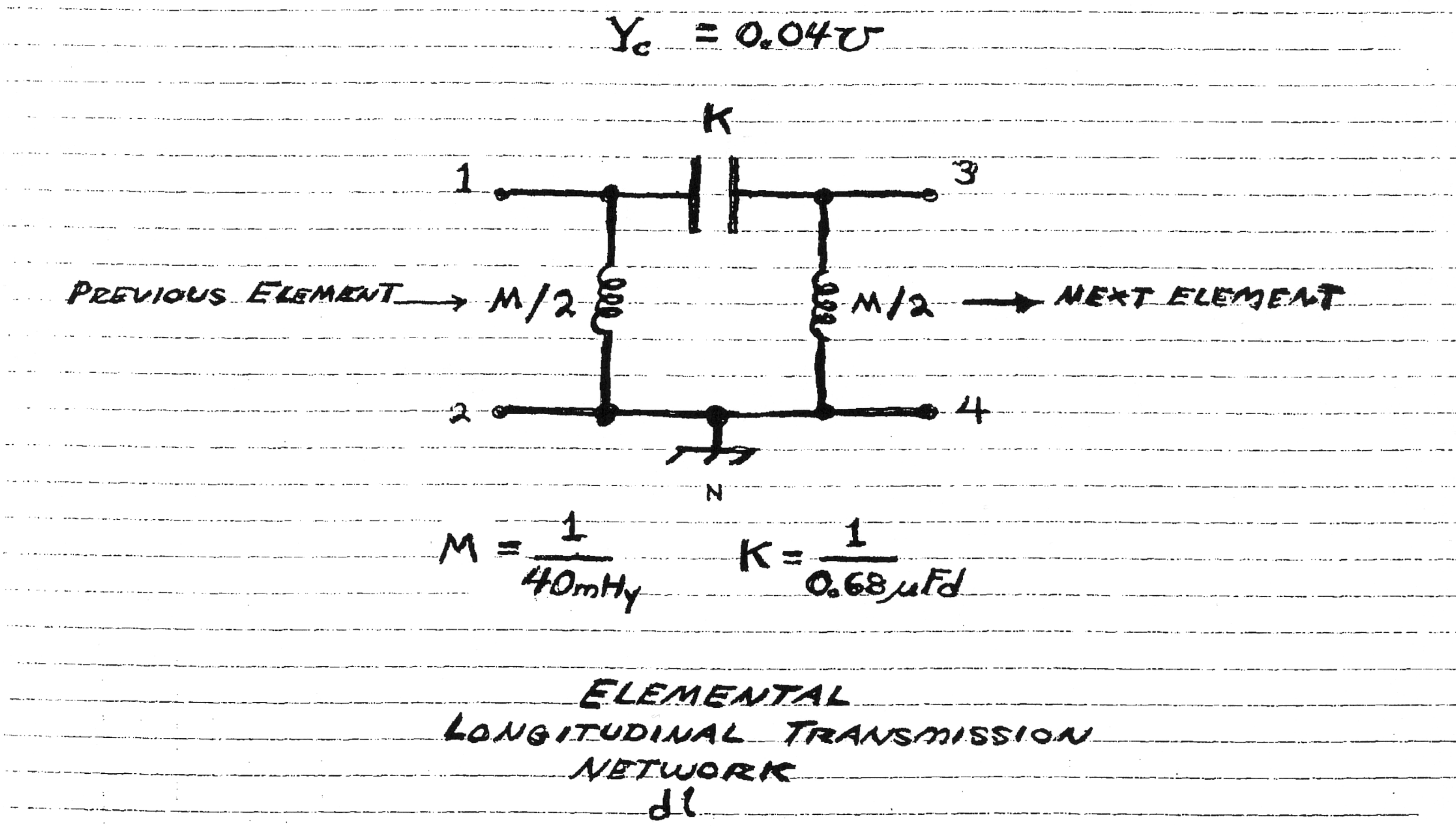
a) CAREFULL ENGINEERING OF CONCATENATED ANALOG NETWORKS GIVE APPEARANCE OF ACTING AS MODULATORS, OR NON LINEAR TRANSMISSION STRUCTURES, THESE STRUCTURES CONSISTING OF LINEAR ELEMENTS, L, C, M, K.

b) LIKEWISE, PROPERLY ENGINEERED NETWORKS IS CAPABLE OF CREATING COMPLEX MUSICAL TONES FOR SYNTHESIZER USE.

c) UNDERSTANDING GAINED FROM NETWORK STUDY MAY LEAD TO MORE ADVANCED KNOWLEDGE OF TRANSFORMER & WAVEGUIDE STRUCTURES IN THE TRANSMISSION & ~~SP~~ DISTORTION OF ELECTRIC WAVES.

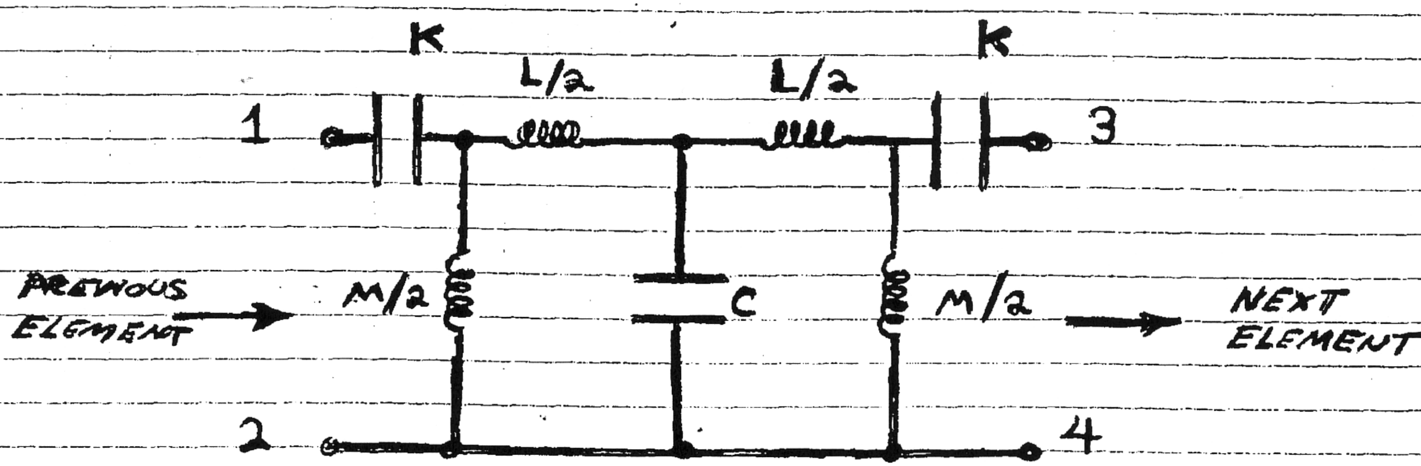


APX I - 1 Elemental Transverse Transmission Network.



APX I – 2 Elemental Longitudinal Transmission Network.

ELEMENTAL SERIES
CONCATENATED TRANSMISSION
NETWORK dl

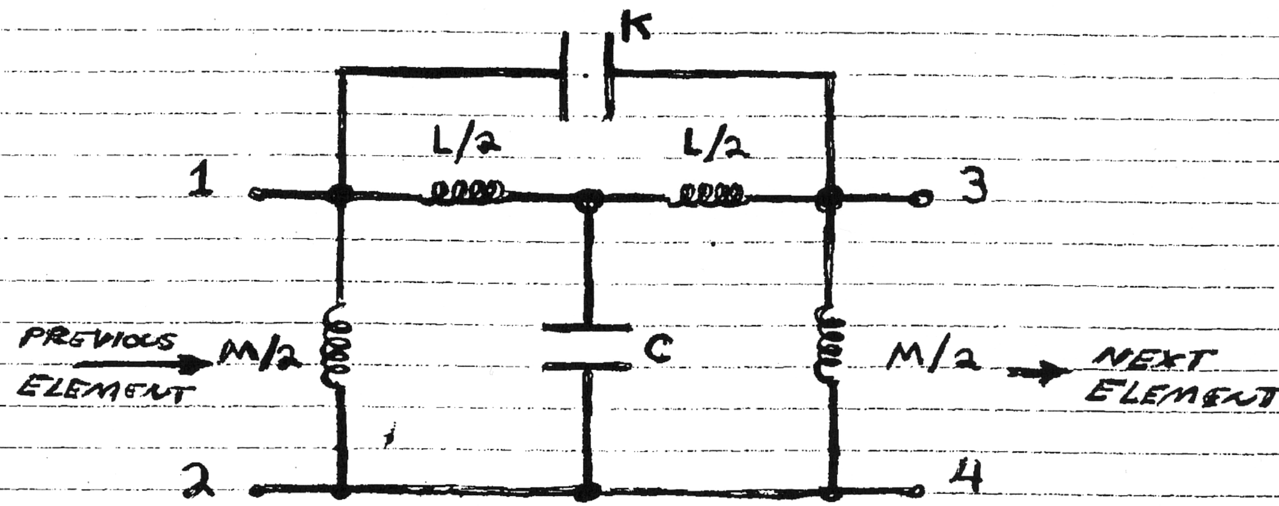


$$C = 1/K = 0.68\ \mu Fd$$

$$L = 1/M = 40\ mHy$$

APX I – 3 Elemental Series Concatenated Transmission Network.

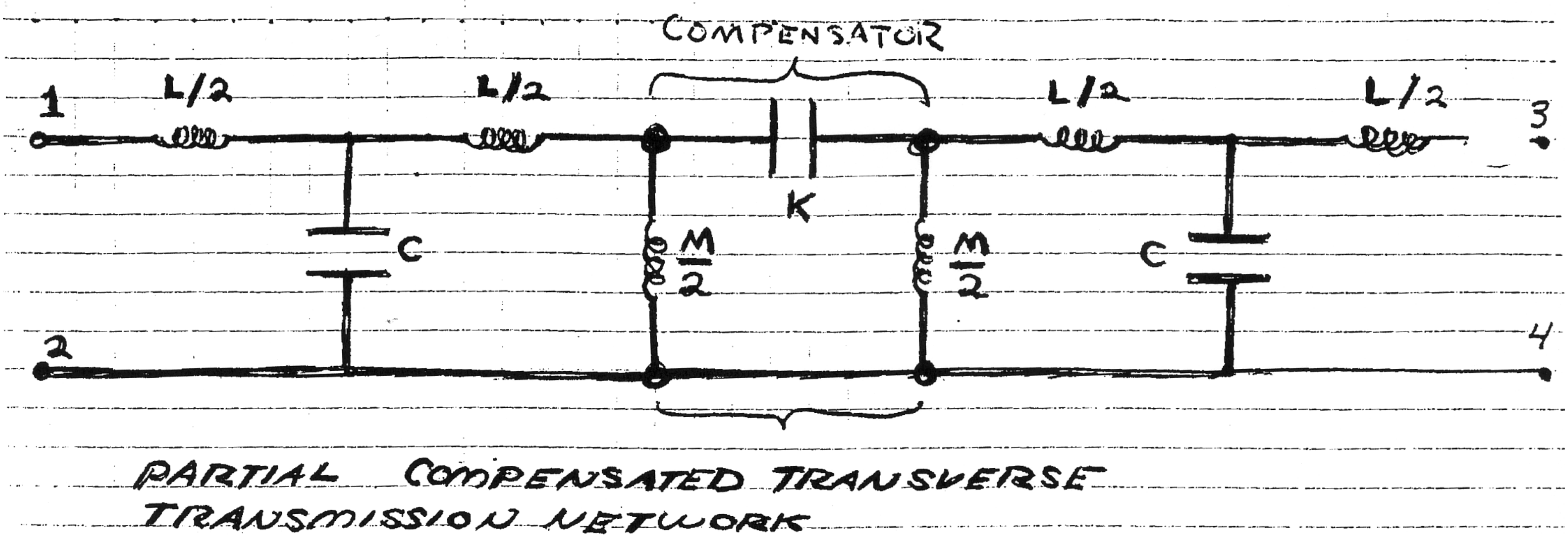
ELEMENTAL SHUNT
CONCATENATED TRANSMISSION
NETWORK dl



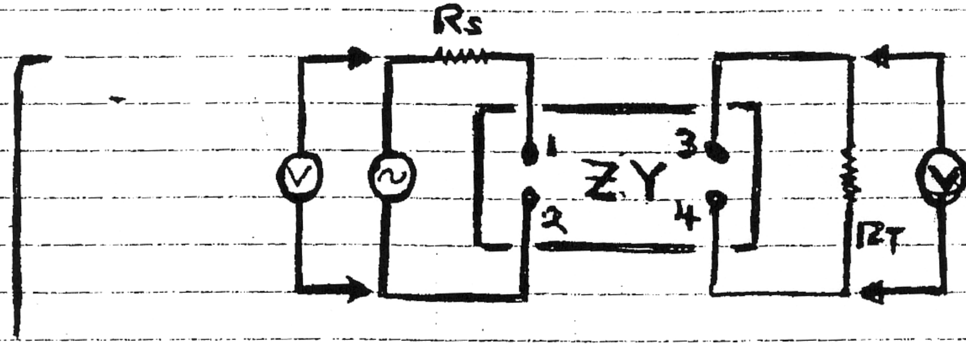
$C = 1/K = 0.68\,\mu Fd$

$L = 1/M = 40\,mH_y$

APX I – 4 Elemental Shunt Concatenated Transmission Network.



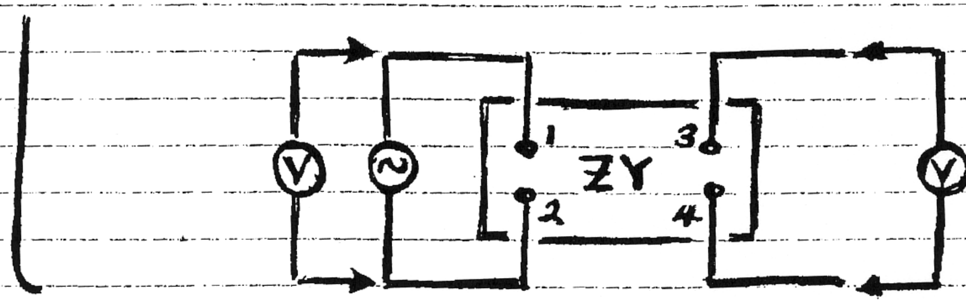
APX I – 5 Partial Compensated Transverse Transmission Network.



TERMINATED TRANSMISSION NETWORK, $R_S = R_T = Z_0$

NETWORK TEST SET UP FOR MEASUREMENT

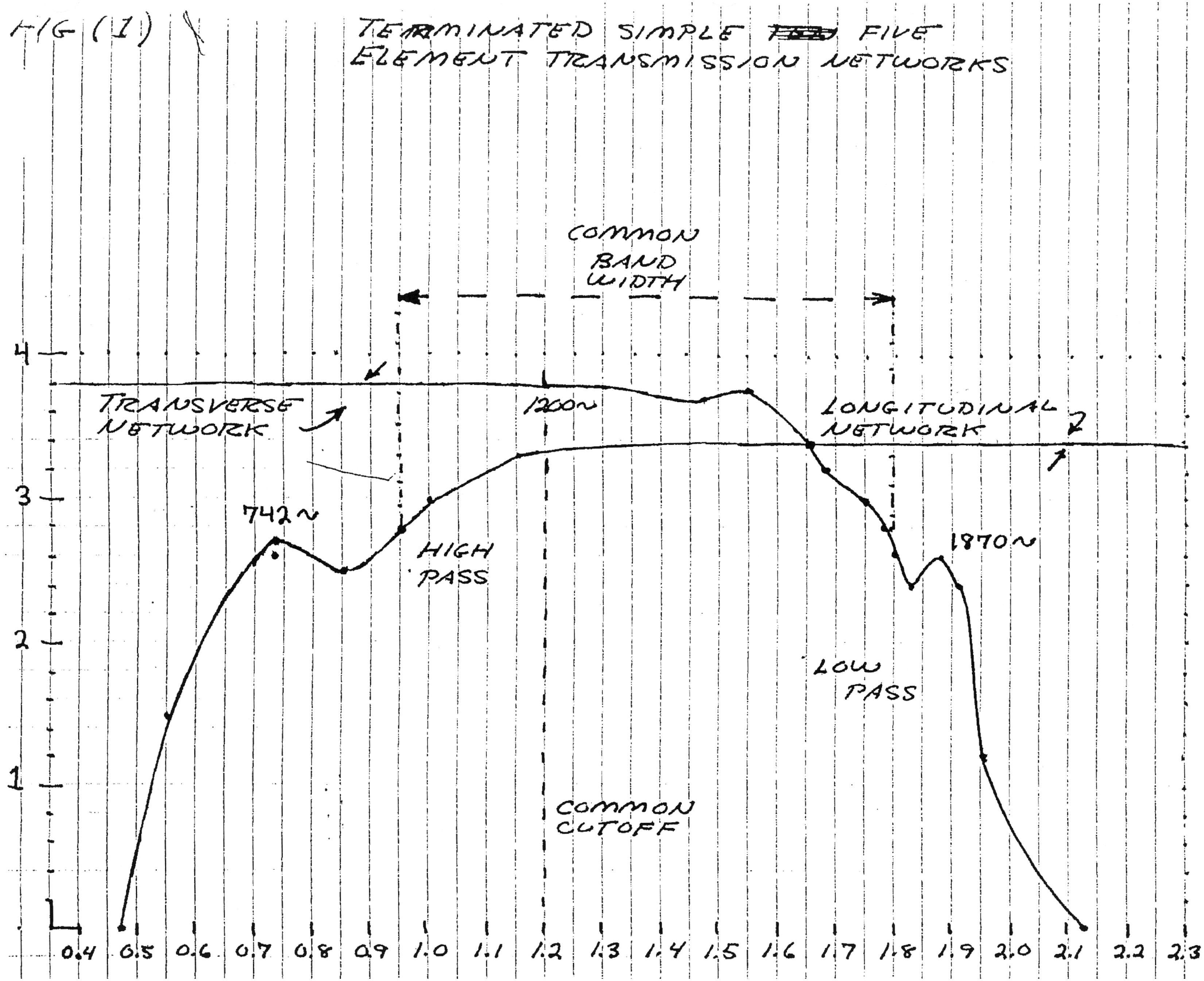
$\frac{Z}{Y} = Z_0^2$



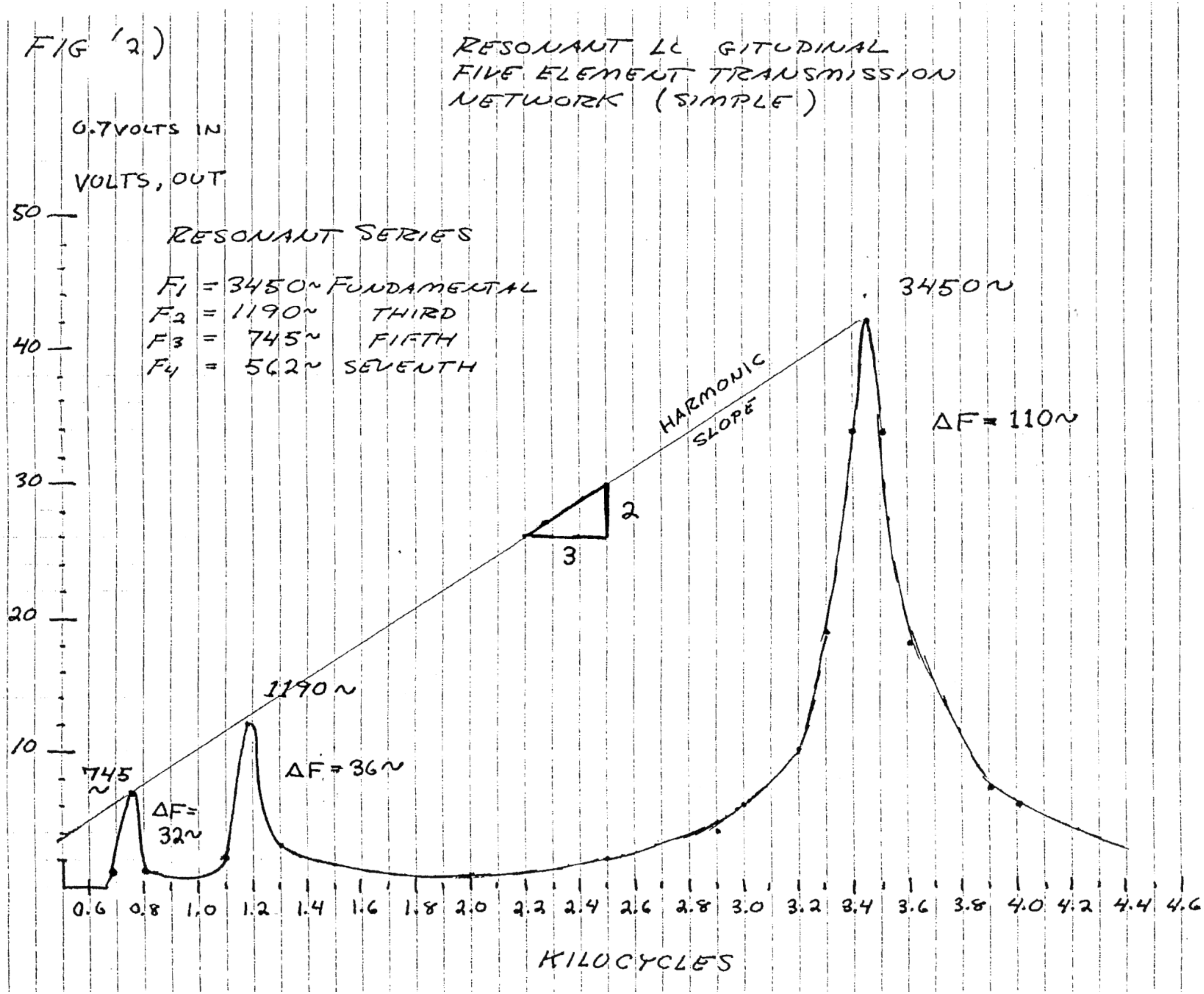
RESONANT TRANSMISSION NETWORK $R_S < Z_0 < R_T$

$R_T = \infty$

APX I – 6 Network Test Setup for Measurement.

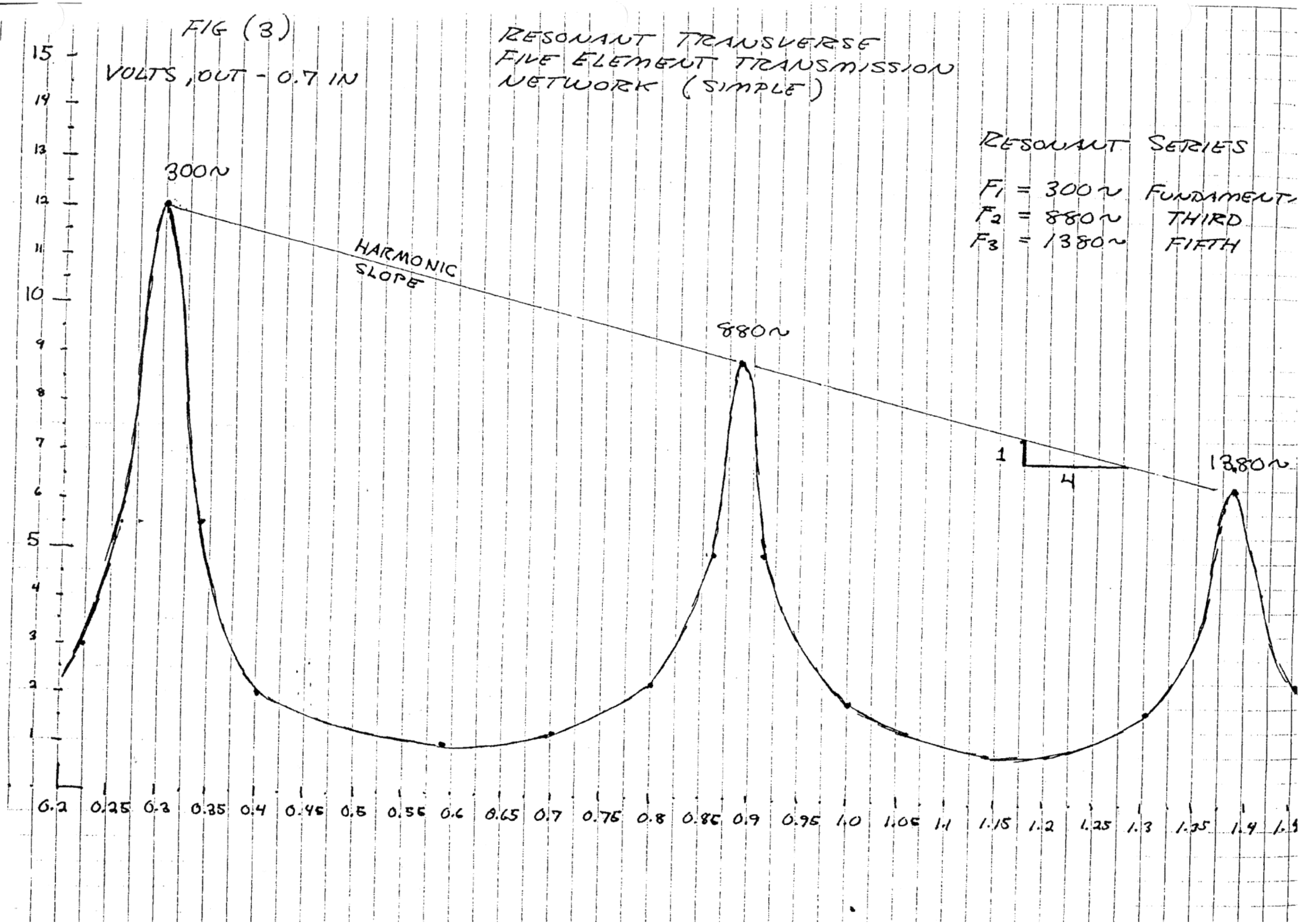


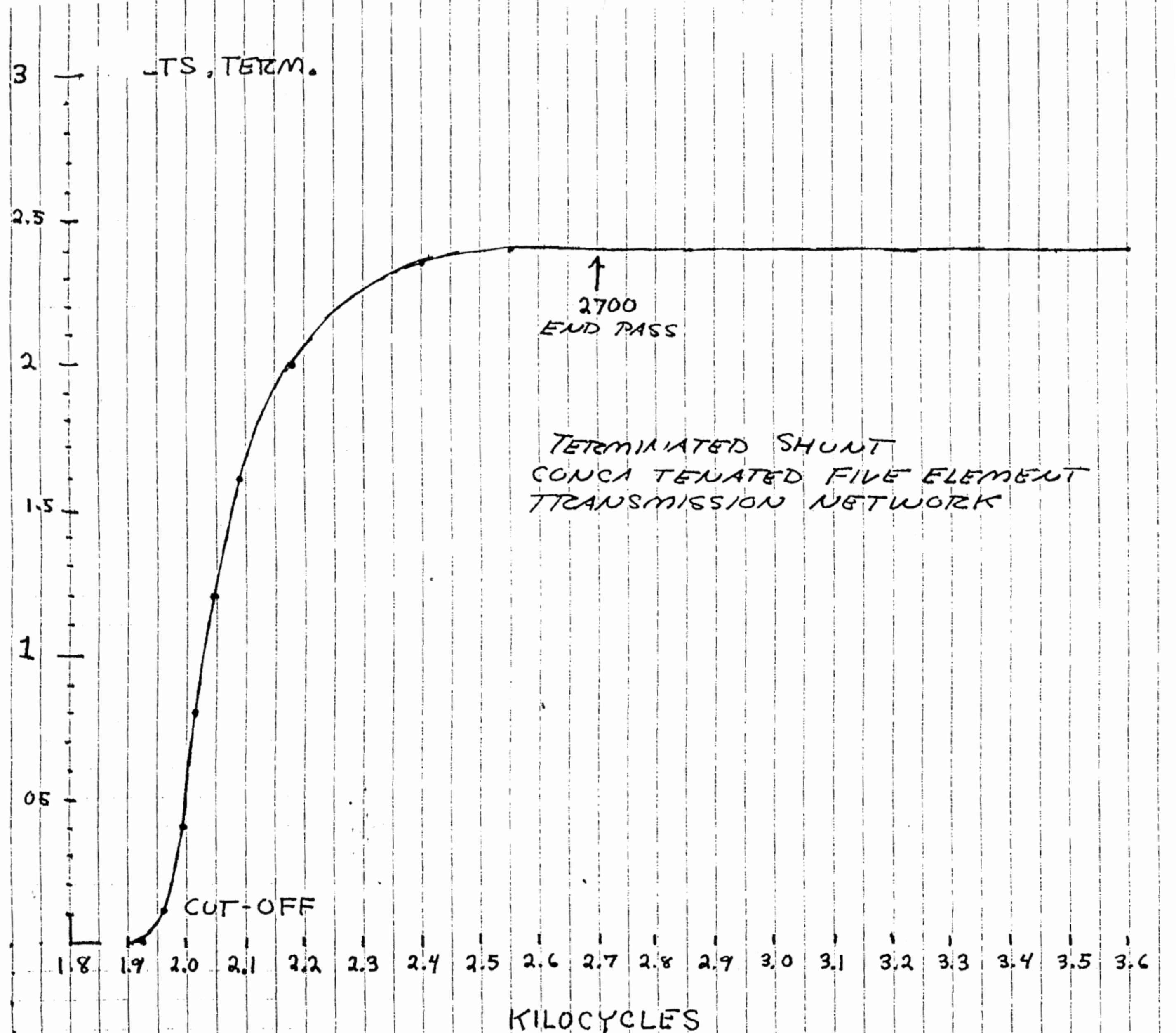
APXI-7 Terminated Simple Five Element Transmission Networks (EPD Fig. 1)



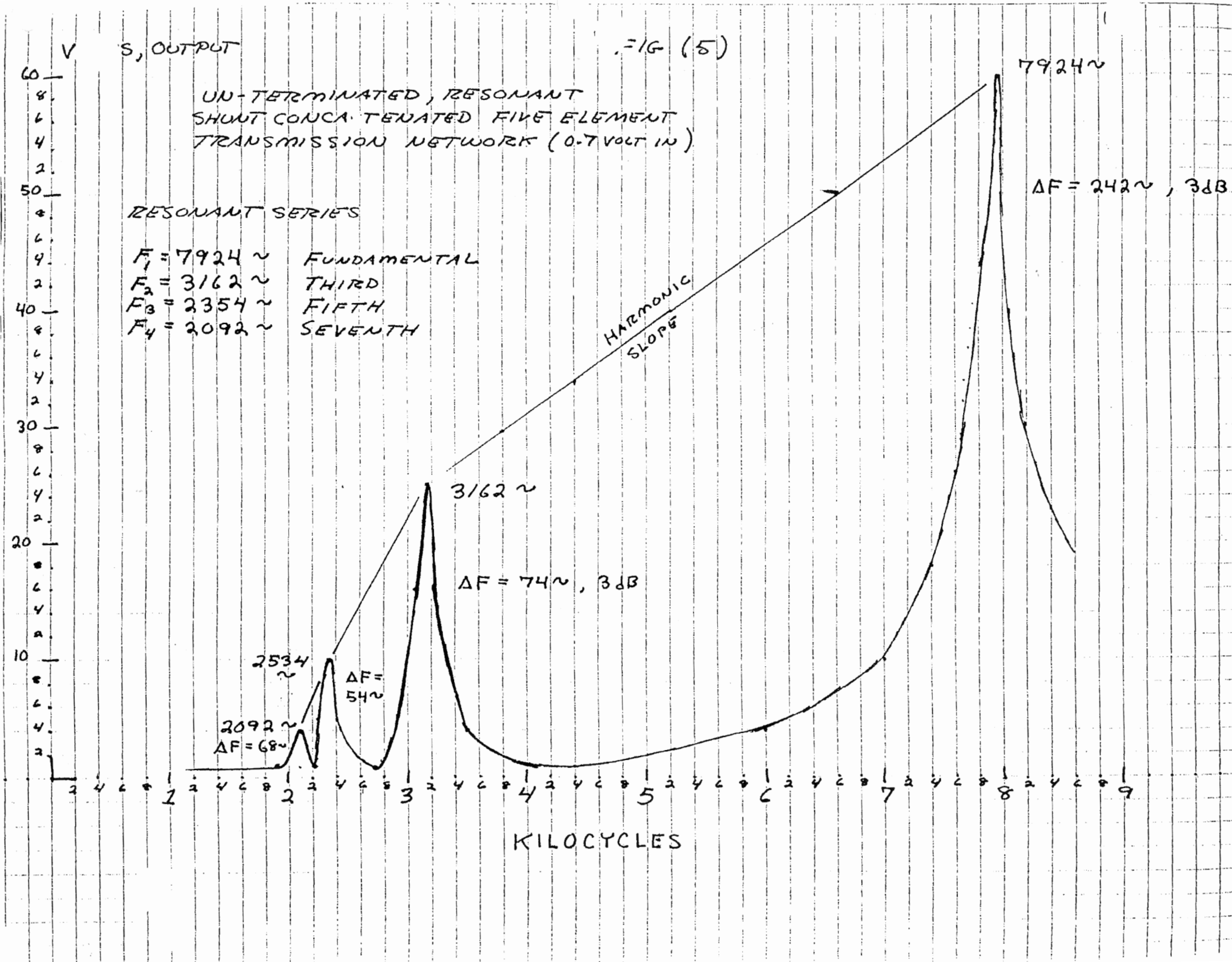
APIX - 8 Resonant Longitudinal Five Element Transmission Network (Simple) (EPD Fig.2)

APX I – 9 Resonant Transverse Five Element Transmission Network (Simple).
(EPD Fig. 3)

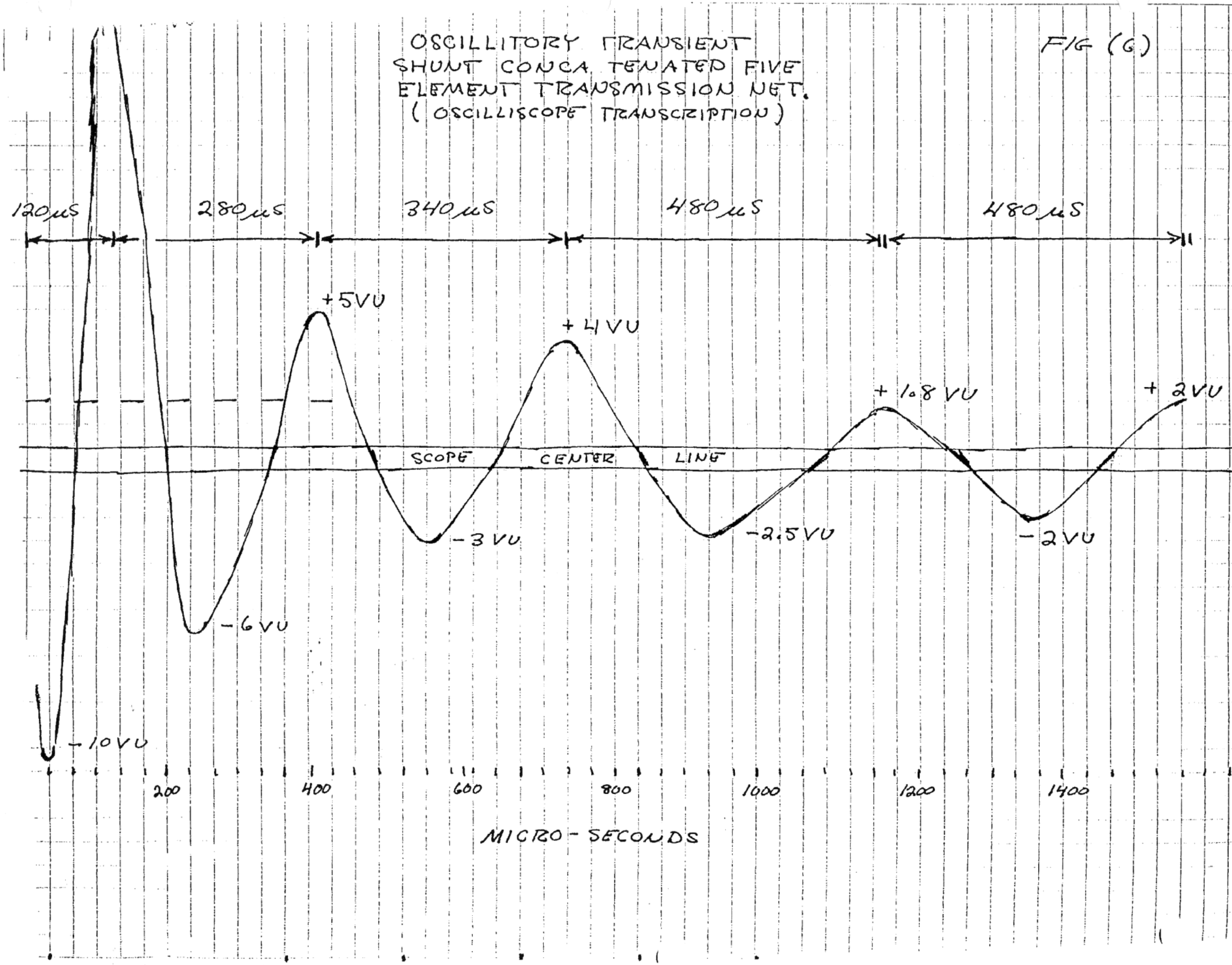




APX I – 10 Terminated Shunt Concatenated Five Element Transmission Network.
(EPD Fig. 4)

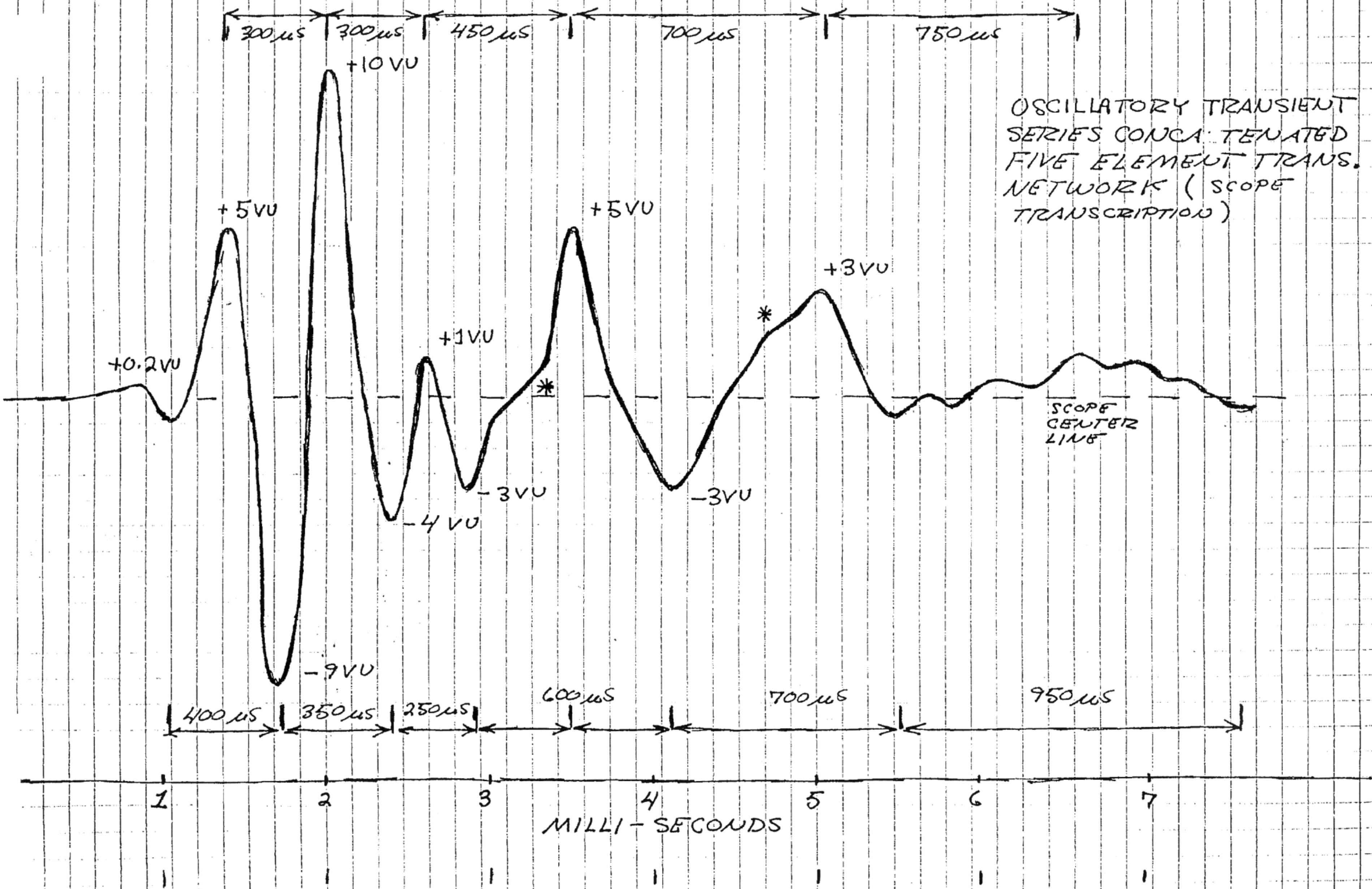


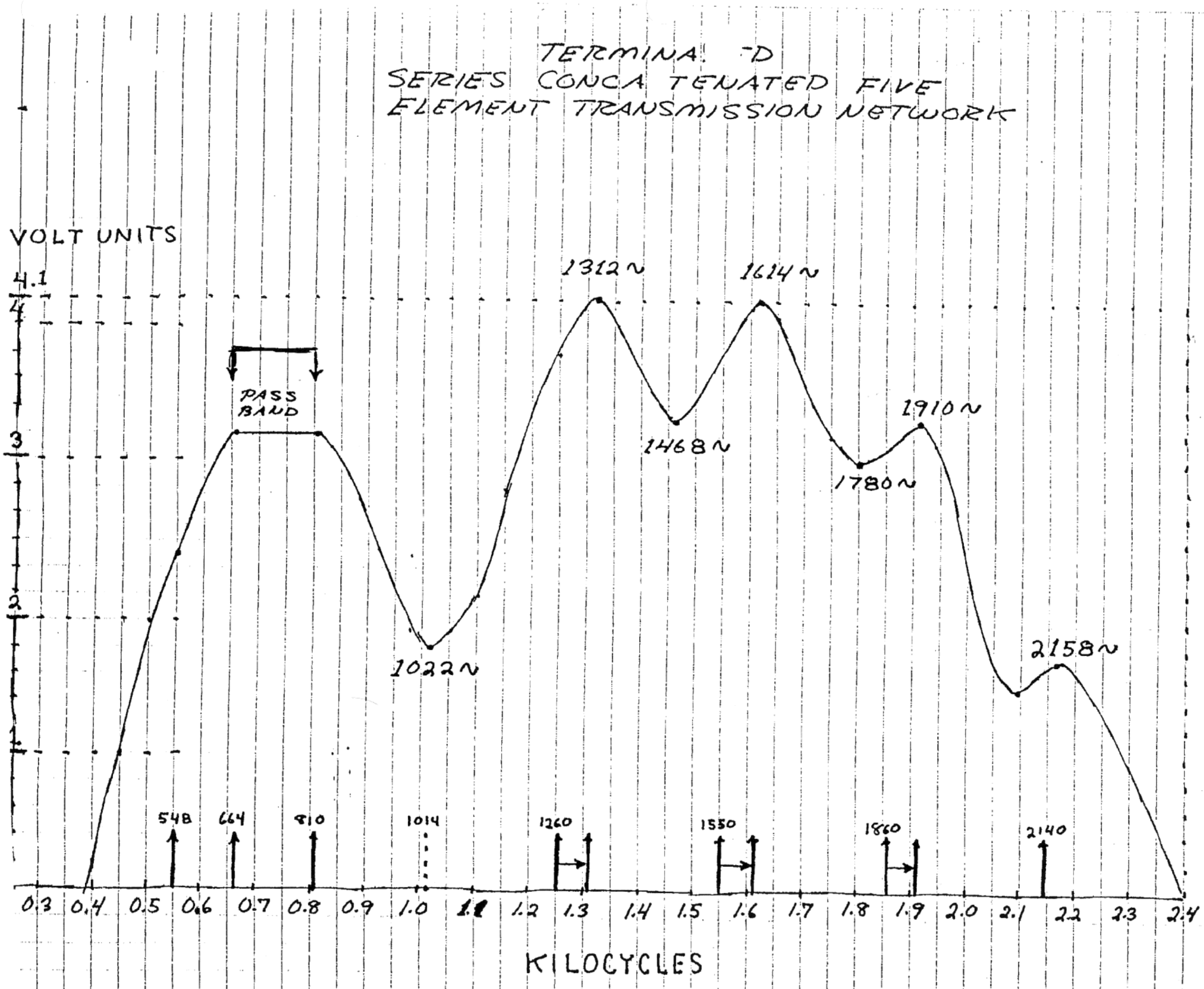
APX I – 11 Un-Terminated, Resonant Shunt Concatenated Five Element Transmission Network (0.7 volt in). (EPD Fig. 5)



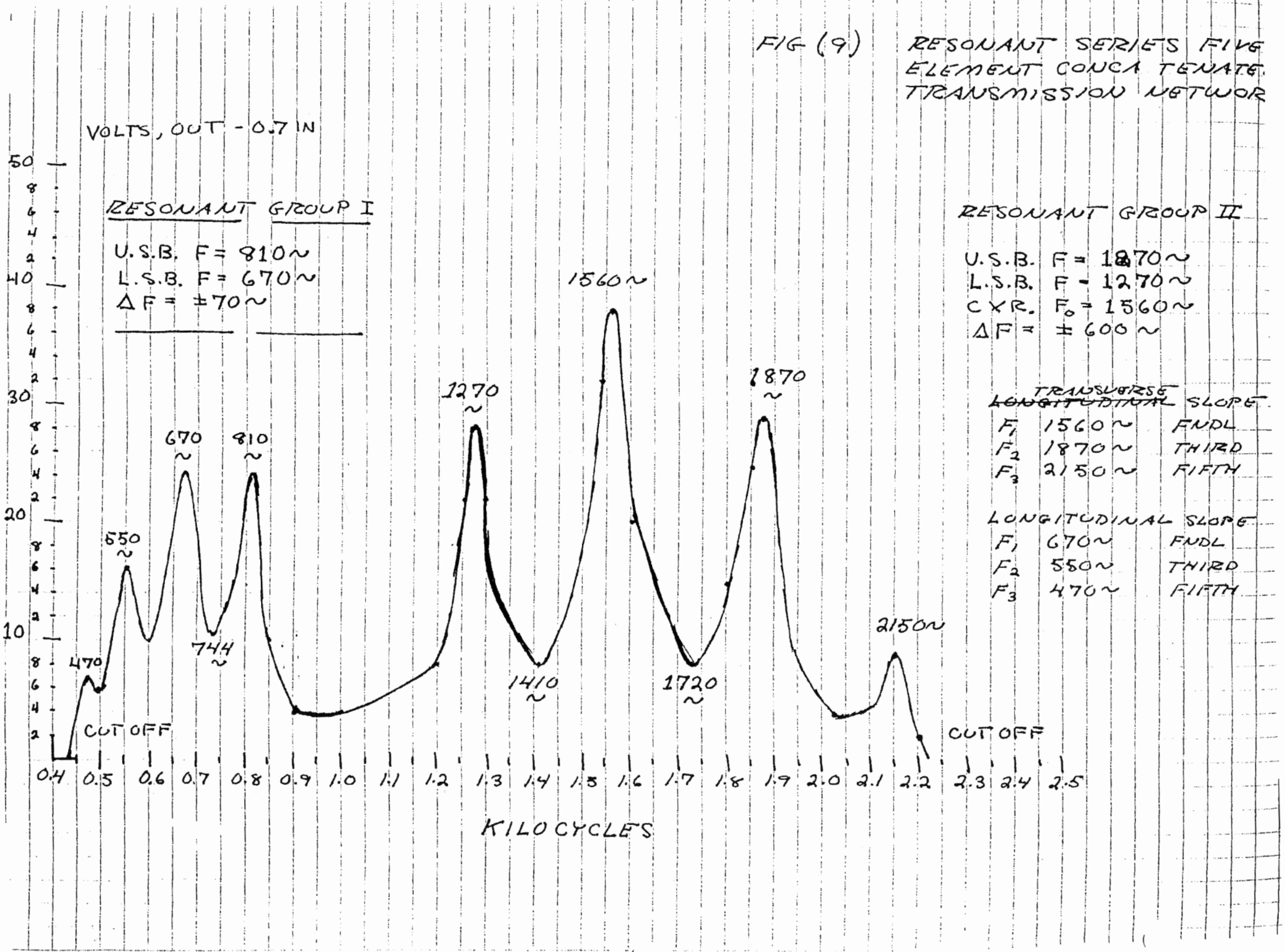
APX I – 12 Oscillatory Transient Shunt Concatenated Five Element Transmission Net (oscilloscope transcription). (EPD Fig. 6)

FIG 7


300 μS
300 μS
450 μS
700 μS
750 μS
+10 VU
OSCILLATORY TRANSIENT
SERIES CONCA TENATED
FIVE ELEMENT TRANS.
NETWORK (SCOPE
TRANSCRIPTION)
+5 VU
+5 VU
+3 VU
*
+1 VU
+0.2 VU
*
SCOPE
CENTER
LINE
-3 VU
-3 VU
-4 VU
-9 VU
400 μS
350 μS
250 μS
600 μS
700 μS
950 μS
1
2
3
4
5
6
7
MILLI-SECONDS



APX I – 14 Terminated Series Concatenated Five Element Transmission Network.



APX I – 15 Resonant Series Five Element Concatenated Transmission Network.
(EPD Fig. 9)

APPENDIX II

IN THE INVESTIGATION OF THE ELECTRIC PHENOMENA THAT OCURR IN RECURRENT ANTENNA STRUCTURES, HIGH FREQUENCY OSCILLATION TRANSFORMERS, OR ELECTRIC WAVE FILTERS, THESE ARE DELT WITH AS LUMPED CONSTANT NETWORKS, THAT IS, AS SYSTEMS OF DESCRETE COILS & CONDENSERS. ANALYSIS IS FROM THE STANDPOINT OF ELECTRO-MAGNETIC CIRCUIT THEORY, BUT IN REALITY THE ELECTRIC CONSTANTS ARE OF A DISTRIBUTED FORM AND THE PHENOMENA IS IN THE FORM OF TRAVELLING WAVES. WHILE THE USE OF TRAVELLING WAVE THEORY IS COMMONPLACE IN THE STUDY OF ELECTRO-MAGNETIC TRANSMISSION LINES IT HAS NOT BEEN DEVELOPED FOR MORE COMPLEX STRUCTURES. THE WRITINGS OF THE PRINCIPLE GENERAL ELECTRIC SCIENTISTS, C.P. STEINMETZ AND L.V. BEWELY, HAVE LAYED IMPORTANT GROUNDWORK BUT FALL SHORT IN ESTABLISHING PRACTICAL FORMULAE. SO FAR NO ONE HAS TAKEN STEPS TO FINALISE THE EFFORTS OF THESE PIONEERS OF ELECTRICAL THEORY. IT IS THE AIM OF THIS PAPER TO INITIATE THE CONTINUATION OF THIS STUDY AND TO DEVELOP A BASIC TRANSMISSION THEORY CONSISTANT WITH THE FOUNDATION LAYED BY OLIVER HEAVISIDE & ARTHUR KENNELY. THRU THIS STUDY NEW CONCEPTS IN RADIO THEORY & PRACTICE AS WELL AS THE GENERATION & REPRODUCTION OF MUSICAL WAVE FORMS WILL RESULT. THE RECENT STRIVING FOR NEW TYPES OF VACUUM TUBE AUDIO EQUIPMENT AS WELL AS RENEWED INTEREST IN THE WORKS OF EGNIMATIC VICTORIAN PHILOSOPHERS SUCH AS TESLA, & EVEN KEELY HAVE GIVEN IMPETUS FOR ADVANCEMENT.

THE PRINCIPLE OBSTACAL IN THIS STUDY IS THE QUADRUPLE ENERGY STORAGE CHARACTER OF THESE NETWORKS. THIS LEADS TO FOURTH ORDER DIFFERENTIAL EQUATIONS WHICH FIND NO SOLUTION IN OUR LIMITED SYSTEM OF ALGEBRA. A SIMPLE PAIR OF COUPLED TUNED CIRCUITS DEMONSTRATES THIS PROBLEM, AND APPROXIMATION OR CONTRIVANCE MUST BE RESORTED TO FOR ANALYSIS. THRU ANALOGY & SYMBOLIC REPRESENTATION THE ARCHETYPAL TELEGRAPH EQUATION OF HEAVISIDE, & THE ANALOG COMPUTING NETWORKS OF KENNELY CAN BE ADAPTED TO A QUANTIFICATION OF WAVE COMPLEXES. THE FOLLOWING MATHEMATICAL PROPOSITIONS SHOULD IN NO WAY BE CONSTRUED AS FINALISED SOLUTIONS BUT ARE GIVEN AS VISUALIZATION AIDS TO THE STUDY OF THE INVOLVED PROCESSES.

LET THE GENERALIZED ELECTRIC WAVE PROPAGATION BE REPRESENTED BY THE FOLLOWING FIGURE;

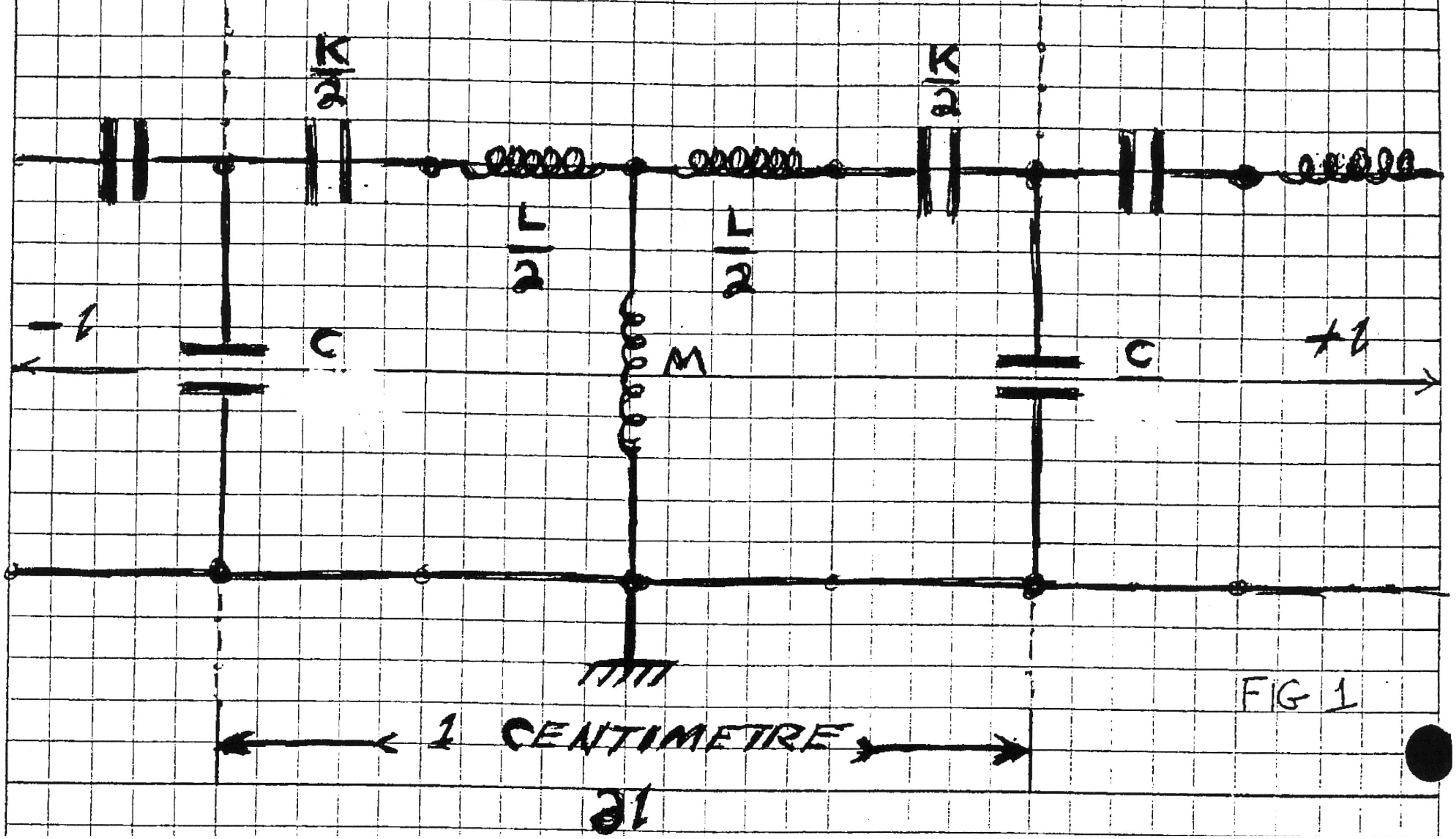


FIG 1

WHERE

C IS THE COEFFICIENT OF DIELECTRIC INDUCTION TRANSVERSE TO THE DIRECTION OF PROPAGATION IN FARADS PER CENTIMETRE,

K IS THE COEFFICIENT OF DIELECTRIC INDUCTION LONGITUDINAL WITH THE DIRECTION OF PROPAGATION IN PER FARAD CENTIMETRE,

L IS THE COEFFICIENT OF MAGNETIC INDUCTION TRANSVERSE TO THE DIRECTION OF PROPAGATION IN HENRYS PER CENTIMETRE,

M IS THE COEFFICIENT OF MAGNETIC INDUCTION LONGITUDINAL WITH THE DIRECTION OF PROPAGATION IN PER HENRY CENTIMETRE

THESE COEFFICIENTS ARE DEFINED BY THE ESTABLISHED CONVENTIONAL PHYSICAL DIMENSIONS,

$$C = \frac{t^2}{l^3} \quad \text{SEC}^2 \text{ PER CM}^3 \qquad (1)$$

$$K = \frac{l}{t^2} \quad \text{CM PER SEC}^2 \qquad (2)$$

$$L = l \quad \text{CM} \qquad (3)$$

$$M = \frac{1}{l^3} \quad \text{PER CM}^3 \qquad (4)$$

These coefficients are defined by the established conventional physical dimensions

$$C = \frac{t^2}{l^3}$$ sec^2 per cm^3 (1)

$$K = \frac{l}{t^2}$$ cm per sec^2 (2)

$$L = l$$ cm (3)

$$M = \frac{1}{l^3}$$ per cm^3 (4)

LET THE SHUNT COEFFICIENTS BE COMBINED BY THE RELATION

$$M + \mathcal{U}_{I}^{2} C = M + \frac{1}{(k\omega)^2} C \qquad (5)$$

AND LET THE SERIES COEFFICIENTS BE COMBINED BY THE RELATION

$$K - \mathcal{U}_{II}^{2} L = K - \frac{1}{(k\omega)^2} L \qquad (6)$$

WHERE THE FACTOR

$$\mathcal{U}^{-2} = (k\omega)^{+2} \quad \text{PER SEC}^2 \qquad (7)$$

AND

$$\omega = 2\pi F \quad \text{PER SEC}$$

$$k^4 = -1 \text{ , A DIMENSIONLESS UNIT.}$$

$$k^4 = -1$$

Let the shunt coefficients be combined by the relation

$$M + u^2 C = M + \frac{1}{(k\omega)^2} C \qquad (5)$$

And let the series coefficients be combined by the relation

$$K - u^2 L = K - \frac{1}{(k\omega)^2} \qquad (6)$$

Where the factor

$$u^{-2} = (k\omega)^{+2}$$ per second^2 (7)

and

$$\omega = 2\pi F$$ per second

$$k^4 = -1$$ A dimensionless unit

THE PRODUCT OF EQ (5) & EQ (6) GIVES THE COMPLETE ALGEBRAIC EXPRESSION OF THE ELEMENTAL SECTION IN FIGURE (1)

$$\left(M+u_{I}^{2}C\right)\left(K-u_{II}^{2}L\right)=\Gamma^{4} \qquad (8)$$

AND CARRYING THRU THE PRODUCTS GIVES THE RELATION

$$\Gamma^{4}=\left[MK+\left(u_{I}^{2}u_{II}^{2}\right)LC\right]+\left[u_{I}^{2}CK-u_{II}^{2}LM\right] \qquad (9)$$

EQ (9) REPRESENTS THE FOURTH ORDER DIFFERENTIAL EQUATION OF THE COMPLEX PROPAGATION THRU THE ELEMENT, FIG (1).

IT WILL BE SEEN THAT EQ (9) IS DIRECTLY ANALOGOUS WITH THE HEAVISIDE TELEGRAPH EQUATION,

$$(R + u_0 L)(G - u_0 C) = \Gamma^2$$

$$= (RG + u_0^2 LC) + u_0 (LG - RC) \qquad (10)$$

WHERE

R IS THE SERIES RESISTANCE IN OHMS PER CM

AND

G IS THE SHUNT CONDUCTANCE IN PER OHM CM

THE FOUR COMPONENTS OF EQ (9) ARE THUS,

I) **MK** REPRESENTS THE LONGITUDINAL WAVE OF ELECTRIC INDUCTION, AND BY EQ (2), (4) IT IS DEFINED

$$MK = 1/l^2t^2 \quad \text{, PER } CM^2 SEC^2 \tag{11}$$

II) **LC** REPRESENTS THE TRANSVERSE WAVE OF ELECTRIC INDUCTION, AND BY EQ (1), (3) IT IS DEFINED

$$LC = t^2/l^2 \quad \text{, } SEC^2 \text{ PER } CM^2 \tag{12}$$

III) **CK** REPRESENTS THE DISTRIBUTION OF DIELECTRIC INDUCTION, AND BY EQ (1), (2) IT IS DEFINED

$$\mathbf{CK} = 1/t^2 \quad , \quad \text{PER CM}^2 \tag{13}$$

IV) **LM** REPRESENTS THE DISTRIBUTION OF MAGNETIC INDUCTION, AND BY EQ (3), (4) IT IS DEFINED

$$\mathbf{LM} = 1/t^2 \quad , \quad \text{PER CM}^2 \tag{14}$$

THE INTERACTION OF THE LONGITUDINAL WAVE (MK) WITH THE TRANSVERSE WAVE (LC) IS REPRESENTED BY THE FACTORS ω_I^2 & ω_{II}^2 WHERE,

$L/K = \omega_I^{-2}$ REPRESENTS THE TIME CONSTANT OF THE SERIES COEFFICIENTS AND BY EQ (2), (3) IT IS DEFINED

$$L/K = \text{PER SEC}^{-2} \tag{15}$$

$C/M = \omega_{II}^{-2}$ REPRESENTS THE TIME CONSTANT OF THE SHUNT COEFFICIENTS AND BY EQ (1), (4)

$C/M = \text{PER SEC}^{-2}$ (16)

THE FACTOR (MK) REPRESENTS WAVE PROPAGATION THRU COUNTER SPATIAL DISTANCE, PER CM. OVER TIME PERIOD, SEC.

THE FACTOR (LC) REPRESENTS WAVE PROPAGATION THRU SPATIAL DISTANCE, CM. OVER TIME PERIOD, SEC.

THE FACTOR (CK) DOES NOT PROPAGATE, BUT IS A DISTRIBUTION OVER DISTANCE CM, AND LIKEWISE LM DOES NOT PROPAGATE, BUT IS A DISTRIBUTION OVER DISTANCE CM. THESE FACTORS ARE TIME SCALARS, THAT IS, NO VARIATION EXISTS IN THE DIMENSION OF TIME.

THE FACTOR (L/K) OSCILLATES WITH NO VARIATION OVER DISTANCE CM., AND LIKEWISE (C/M) OSCILLATES WITH NO VARIATION OVER DISTANCE. THESE FACTORS ARE FREQUENCIES, OR TIME FACTORS OF SEC^{-1} AND ARE SPACE SCALARS.

IN TERMS OF CIRCUIT ELEMENTS THESE TERMS ARE REPRESENTED BY

MK

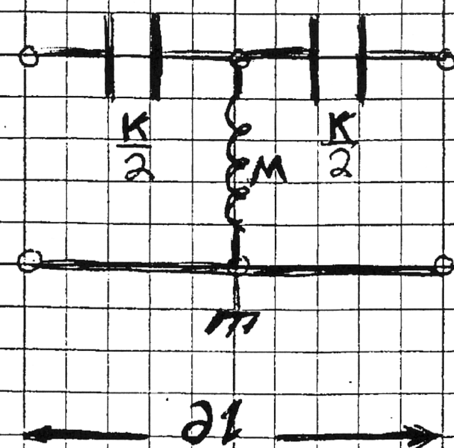


FIG (2)

LC

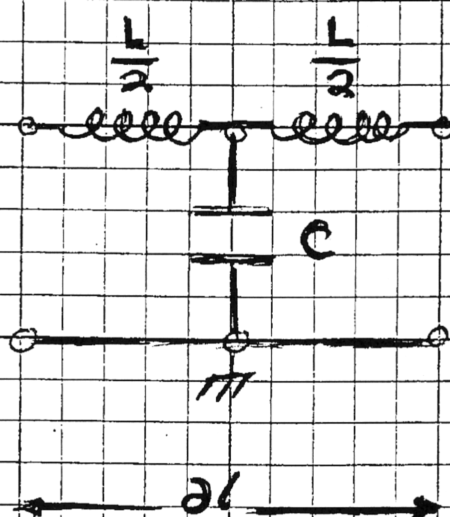


FIG (3)

(MK) CAN BE SEEN AS A FUNDAMENTAL HIGH PASS SECTION OR PHASE LEAD NETWORK, AND BECOMES TRANSPARENT TO PROPAGATION FOR

$$\partial/\partial t \longrightarrow \infty$$

(LC) CAN BE SEEN AS A FUNDAMENTAL LOW PASS SECTION OR PHASE LAG NETWORK, AND BECOMES TRANSPARENT TO PROPAGATION FOR

$$\partial/\partial t \longrightarrow 0$$

LIKEWISE

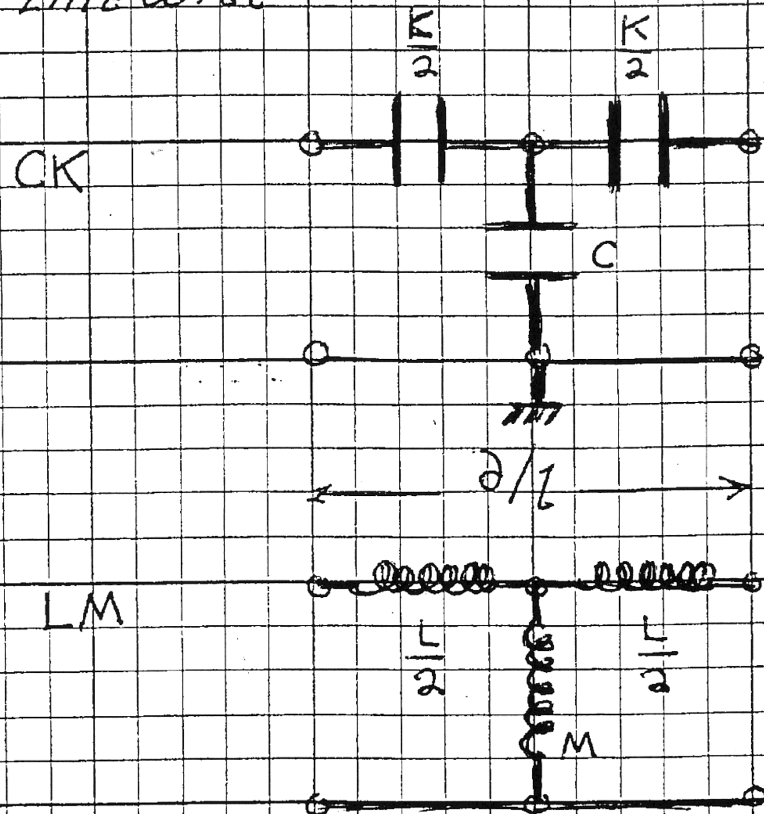


FIG (4)

WHERE (CK) & (LM) DO NOT PROPAGATE BUT ARE INSTANTANEOUS DISTRIBUTIONS OF THE DIELECTRIC & MAGNETIC FIELDS OF INDUCTION RESPECTIVLY. INSTANTANEOUS ACTION IS CARRIED THRU K IN THE DIELECTRIC DISTRIBUTION & M IN THE MAGNETIC DISTRIBUTION. AND FINALLY

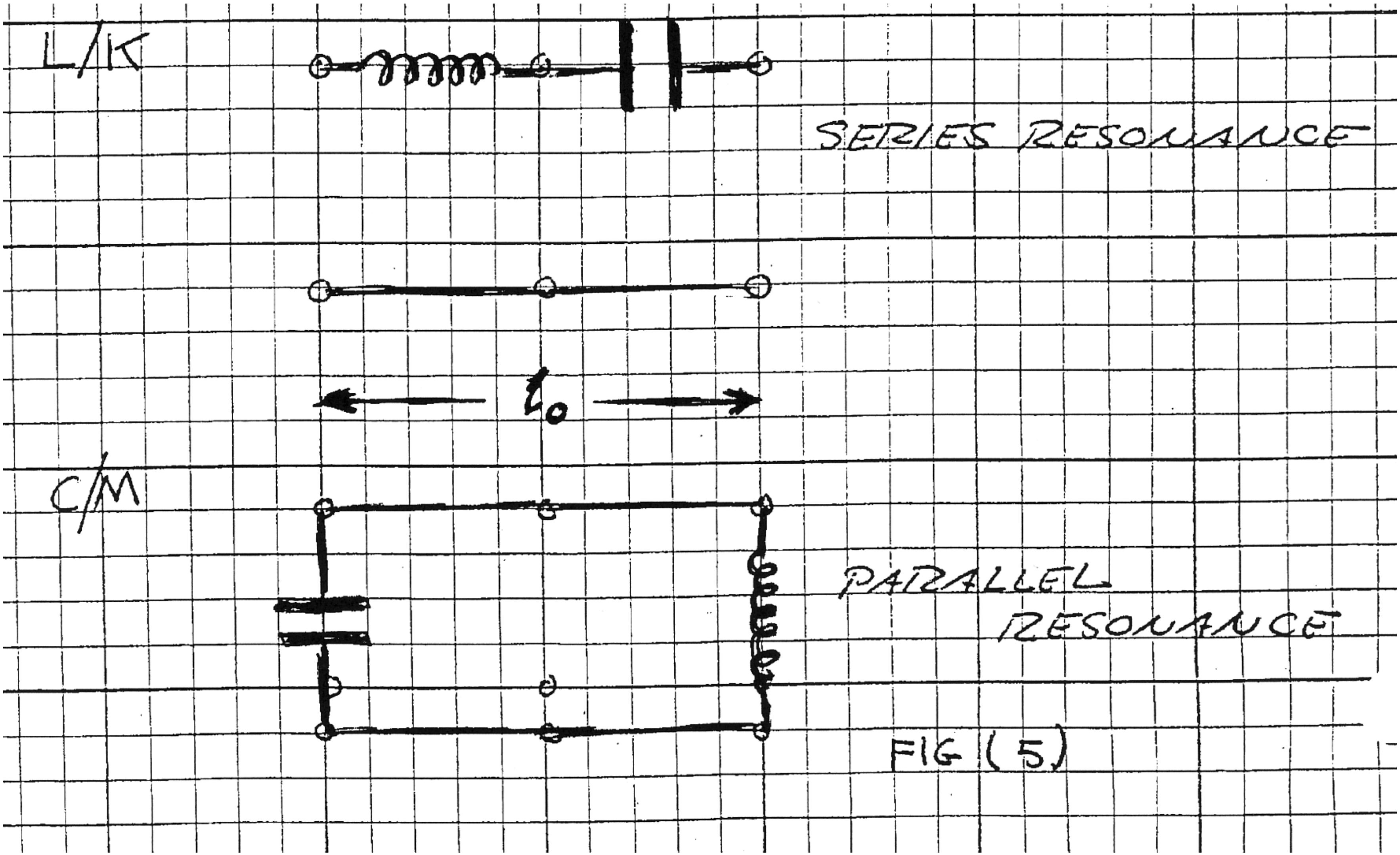


FIG (5)

WHERE (L/K) & (C/M) DO NOT PROPAGATE BUT ARE THE SERIES & PARALLEL RESONANT FREQUENCIES OF THE ENTIRE ELECTRICAL NETWORK OF LENGTH l_0.

IN GENERAL ANY ELECTRICAL NETWORK OF THE FORM IN FIG (1) WILL EXIBIT A PAIR OF FREQUENCIES WHICH MAY BE DISSONANT OR CONSONANT DEPENDING ON THE RELATION BETWEEN (LM) AND (CK). IF SUCH A NETWORK IS TERMINATED IN ITS IMAGE IMPEDANCE & ADMITTANCE IT WILL EXHIBIT A BAND PASS CHARACTERISTIC PASSING A DEFINITE WINDOW OF FREQUENCIES.

THE FREQUENCIES WILL BE IN UNISON IF THE CONDITION

$$L/K = C/M \tag{17}$$

EXISTS, OR TRANSPOSING,

$$LM = CK \tag{18}$$

THAT IS, IF THE DISTRIBUTION OF MAGNETIC INDUCTION EXACTLY MATCHES THE DISTRIBUTION OF DIELECTRIC INDUCTION, THE NETWORK IS THEN DISTORTIONLESS.

CONVERSELY, IF (L/K) OR (C/M) IS MADE TO VANISH, LEAVING ONLY ONE OR THE OTHER, SUCH AS WITH CADUCEOUS WOUND NETWORKS OR LUMPED CONSTANTS ONLY A SINGLE FREQUENCY EXISTS AND THE NETWORK IS SPACE SCALAR.

III General Theory of Telluric Electric Wave Transmission and Reception

(1) Introduction

(a) The reception and transmission of telluric electric waves employs methods and principles unlike those found in conventional electro-magnetic wave systems. Electric wave propagation within the interior of the earth is of a character much different than that propagation in the space exterior to the solid mass of the earth. The space within the mass of the earth is complicated by various degrees of magnetic permeability, dielectric permittivity, conductivity and resistivity, all these of various magnitudes for various directions. Further is the presence of significant static magnetic and static dielectric fields of induction.

(b) With conventional electro-magnetic structures the principles of wave reception are equivalent to the principles of wave transmission, this is known as the law of reciprocity. In part this law can be applied to the various sub-systems that form the telluric wave systems. However, certain antenna configurations depart from the reciprocity law in that the propagation factor for reception must lag in phase behind the electric wave to be received, where as the propagation factor for transmission must lead in phase ahead of the electric wave to be transmitted. This situation is analogous to the alternating current induction machine. For a given frequency in radians per sec. of electric excitation to the field of the induction machine the frequency in radians per second of rotation must fall behind the excitation frequency for the induction machine to receive energy as a motor. Conversely the frequency of rotation must push ahead of the excitation frequency for the induction machine to transmit energy as a generator. In this situation the angular frequency of excitation is of unit value or represents a relative condition of rest. The angular frequency of rotation has a relative negative or positive, value for positive or negative power flow respectively. For the antenna the situation is the same. The lagging velocity for receive and the leading velocity for transmit, this relative to the velocity of the electric wave in the medium of transmission or reception.

(c) An important condition for the transmission and reception of telluric electric waves is a single wire or uni-polar connection to the solid mass of the earth. Electro-magnetic transmission and reception requires a multi-polar or multiple wire connection, two wire being common. It is required for telluric wave operation that the antenna sub-system be self referencing, that is the antenna sub-system not require grounding in the usual sense, since ground is now an active terminal. There can be no second wire since there is nothing to connect it to. Hence, the need for a single wire or unipolar antenna characteristic.

(d) In the transmission and reception of telluric electric waves two departures exist with regard to the transmission and reception of electro-magnetic waves:

1) The law of reciprocity is not applicable to the transfer of energy between the telluric wave and the antenna sub-system.

2) The boundary condition or circuit law is not applicable to the connection of the antenna to the solid mass of the earth.

(e) Such electrical conditions, once common in early wireless development, have become largely unknown. Two principal systems emerged from this era having the proper characteristics for telluric electric wave applications:

1) The oscillation transformer as developed by Nikola Tesla, 1900

2) The multiple loaded aerial as developed by Ernst Alexanderson, 1919

(2) The Oscillation Transformer

(a) The first development in the wireless transmission of electric waves was a telluric system based upon the application of an antenna sub-system known as the oscillation transformer. This transformer is a single winding coupled magnetically to an external resonant structure. Transformer operation resembles a constant current or ballast transformer. The single winding of the oscillation transformer resembles that of a simple reactance coil, however, only a single lead exists for communicating energy in and out of this coil structure. It is a single wire, uni-polar connection. The second lead of the coil is only connected to a small free space electro-static condenser.

(b) In the operation of the oscillation transformer the winding is not a simple reactance coil and magnetic field of induction. The dielectric field of induction now plays an important role, as energy now resides in the dielectric field in addition to energy residing in the magnetic field. In oscillation transformer operation the total energy divides evenly between the magnetic field and dielectric field of induction. The superposition of these two fields of induction give rise to complex electric waves. The oscillation transformer winding, thus operates as a wave guide structure, giving rise to electric waves through the exchange of magnetic and dielectric energy.

(c) Complex electric standing waves exist on the oscillation transformer winding during its resonant exchange of energy between the two fields. These standing waves produce a phase displacement in the <u>time</u> cycle of energy exchange and in <u>length</u> along the winding structure. The displacements exist as a hysteresis cycle, displacing the cause-effect relationship. This results in the transformer winding operating as a uni-polar system.

(d) The resonant structure coupled to the oscillation transformer winding is a simple magnetic reactance coil in a resonant relation with a simple electrostatic condenser. This circuit is proportioned to have minimal dissipative losses, that is it has a large magnification factor. This circuit provides the two wire connection for the supply or abstraction of energy to or from the oscillation transformer winding and its uni-polar connection to the earth.

(e) In conjunction with the coupled resonant circuit the oscillation transformer winding serves as a phase transformer. This phase transformation provides the basis conversion from a multi-phase to a uni-phase connection. This provides the single wire connection for the telluric electrical waves, transforming this to the multiple wire connection to a network sub-system.

(f) The complex electric wave produced by the resonant electric fields of the oscillation transformer winding is analogous to those electric waves, which exist within the interior of the earth. This complex electric wave in the winding is the resultant of the superposition of transverse electric waves of a specific velocity and of longitudinal waves of a specific counter-velocity. This pair of electric waves propagate within the electric field of the winding. This winding can be proportioned to be attuned to the complex electric wave propagation within the interior of the earth. The winding becomes an analog of the specific telluric waves to be transmitted or received.

(g) The transient impulses produced by the oscillation transformer are of analogous form to the transient impulses resulting from telluric wave propagation within the earth. In this manner the oscillation transformer responds as does the network sub-system previously described. The high and low pass functions are a direct result of transformer actions. Hereby the oscillation transformer serves as the network sub-system in addition to serving as the antenna sub-system. Thus the oscillation transformer in itself serves as a system for the transmission and reception of telluric electric waves.

(h) The principal drawback in the application of the oscillation transformer to telluric waves is the inability to respond to a wide range of signal frequencies. Also is the lack of directivity in the spatial distribution of its response. Thus in the application of the oscillation transformer to telluric waves it can be proportioned to respond only to telluric waves of a single frequency and its related harmonic structure, that is to one specific transient electric wave form. Transformer response is to telluric waves from all directions, it having no directional character. This limits the use of the oscillation transformer to specific communication or broadcast, functions and thus prohibits its use for broadband or generalized transmission or reception functions.

(3) The Multiple Loaded Aerial

(a) Following the development of the oscillation transformer was its application to the system of wireless transmission developed by Guglielmo Marconi (1910). During its initial development by Nikola Tesla the wave guide and uni-polar properties of the oscillation transformer were not fully understood. Tesla repeatedly attempted to force the winding to operate as a simple magnetic reactance coil. The importance of the dielectric field of the winding and its complex relation to the magnetic field were to be missed by Tesla and his contemporaries. This situation would be further compounded by the efforts of Marconi.

(b) The application of the oscillation transformer to the transmission and reception of telluric waves was under patent protection by Nikola Tesla. For Marconi to proceed with his wireless development, significant alterations had to be made. In the telluric wave system of Nikola Tesla the oscillation transformer alone served as the basic system for the transmission or reception of electric waves. Marconi would make important changes to the Tesla system in order to secure a wireless patent of his own. The basic modification was the extreme enlargement of the electro-static capacity of the free terminal of the oscillation transformer winding. An aerial-ground structure known as the Marconi "Flat Top", of considerable extent, was connected as a basic condenser to the oscillation transformer winding. The electro-static capacity of this aerial-ground structure greatly exceeded that of the oscillation transformer winding. Hereby the function of the oscillation transformer was reduced to that of a basic magnetic reactance coil. Now unable to resonate with the winding dielectric field, the winding lost the ability to operate as a phase transformer. It now operates as a di-polar or two wire system.

(c) The aerial portion of the Marconi Flat Top was positioned over a similar structure in the ground. The length of this aerial-ground system was several times larger than the width, this forming a large strip-line transmission structure. Thus the aerial-ground structure is a electrically short section of electro-magnetic transmission line. Within the electro-magnetic field of this section of line, a very large reactive power flow exists, this in an oscillitory energy exchange with the transformer. The coil and aerial-ground structure reduce to a basic resonant circuit. The power flow in this circuit contributes little to the ability of the Flat Top to transmit or receive electric waves. The Flat Top derives its ability to transmit or receive principally from it external dielectric field of induction. The lag of phase along the length of the Flat Top produces a small portion of external electro-magnetic activity and resulting waves. Hence, the Marconi Flat Top aerial-ground system is an ineffective structure for both electro-magnetic and telluric electric waves.

(d) The large reactive power flow within the confined portion of the Flat Top represents a useless or parasitic power flow. Its loading upon the oscillation transformer renders the winding a reactance coil. Therefore, the confined electro-magnetic field of induction inhibits the operation of this aerial-ground system in the transmission or reception of telluric electric waves.

(e) While Marconi resorted to simple terminal impedance methods to minimize the effects of this reactive power flow, the basic situation remained unchanged. Ernst Alexanderson, while employed by the General Electric Company and the U.S. Navy (1919), developed a significant advancement in the Flat Top system. This development became the Alexanderson multiple loaded aerial. This aerial-ground system finds important applications to the transmission and reception of telluric electric waves.

(f) The Alexanderson system is a direct adaption of the Marconi Flat Top. The basic external geometry is unchanged. However, the aerial and the ground elements of the strip-line configuration are sectionalized into a series of sub-section elements. Loading elements are inserted in the transitions between sub-section elements. The Alexanderson principle utilized this sequential loading to cancel or neutralize the reactive electro-magnetic power flow of the Flat Top system. The result is the aerial-ground system becomes a non electro-magnetic structure, with the dimensions of velocity and wavelength becoming undefined.

(g) The Alexanderson system is no longer the simple strip-line of Marconi, but has become a complex system of alternate, sequential sections of transmission and loading structures. This configuration is analogous to a loaded long distance telephone line. The Alexanderson system has rendered the strip-line of Marconi to a wave-guide type structure. The superposition of the magnetic field and the dielectric field in this wave-guide give rise to complex electric waves as with the oscillation transformer of Tesla. Hereby the Alexanderson system enables telluric wave transmission and reception.

(h) The basic oscillation transformer winding exists in multiple with the Alexanderson system, each being connected at each of the sequential loading sections. These windings now operate unhampered by reactive power flow. Operation of these phase transformer windings in multiple allows for directional operation, unlike a single unit. In addition, in conjunction with the loading elements, the windings in multiple allows for a band pass characteristic to be established. Hereby the Alexanderson multiple loaded aerial-ground system overcomes the principal limitations of the oscillation transformer system of Tesla, the lack of directivity and bandwidth.

(4) Development of the Alexanderson System for the Propagation of Telluric Waves.

(a) The basic Alexanderson system can be developed further for adaption to the propagation of telluric waves. Alexanderson would follow the path that Marconi followed from the Tesla system. The Alexanderson system was operated as a di-polar configuration for the propagation of electro-magnetic waves. This led to the extinction of the Alexanderson system as well as the systems from which it developed. Advancing the Alexanderson concept one step ahead, while retaining the original uni-polar concept of Tesla, results in an aerial-ground sub-system of perfect adaptability to the transmission and reception of telluric waves.

(b) In the layout of the Alexanderson system upon that of Marconi, the earthed portion of this system basically remained unchanged. This portion continued to operate as a single grounded conductor under the aerial portion of the system. No significant phase or potential differences exist along the continuous earthed portion of the Alexanderson system. Therefore, no electric waves can exist along this length of grounded conductor. This earthed portion of the aerial-ground system acts as a single ground electrode and can propagate telluric waves only in the manner of a single point source, as with the Tesla system. During the period of history during which the Alexanderson system existed, it was considered as a system for the propagation of electro-magnetic waves. This related to the vertical conductors rising from the grounded loading sections to the aerial structure above. The electric current related to this conductor gives rise to the propagation of electro-magnetic waves. However, the energy of these waves exists as a small portion of the total electric wave propagation of the Alexanderson aerial-ground system. Alexanderson as well as Marconi engineers understood that the Flat Top aerial and its adaptation by Alexanderson operated as an antenna for the propagation of <u>electro-static</u>, rather than electro-magnetic waves. Therefore, the Alexanderson aerial operates as a system for the transmission and reception of dielectric waves through its external dielectric field. Part of this dielectric field of induction is directed by the earthed ground structure into the interior of the earth. This induction gives rise to the propagation of telluric waves in a manner similar to that of Tesla.

(c) The Alexanderson aerial-ground system is an advancement upon the Marconi system. The Marconi system is an adaptation of the Tesla oscillation transformer system. The antenna sub-system of the basic system for the propagation of telluric waves represents an advancement upon the system of Tesla, this lacking directivity and bandwidth and represents an advancement upon the system of Marconi/Alexanderson, which primarily propagated waves exterior to the mass of the earth. The advancement upon the Alexanderson system is the <u>elimination of the external propagation</u> of electric waves. Advancement centers upon the <u>earthed</u> portion of the aerial-ground system. Unlike the Alexanderson configuration loading is <u>divided in a balanced fashion</u> between both the aerial and earthed portions of the system. Hereby complex electric waves within the interior of the earth can be developed. The elements of the earthed portion of this system operate each independent of the other with no inter-connection. Each element consists of a vertical section projecting within the mass of the earth. The earthed portion of the aerial-ground structure exists as a sequential row of vertical earthed elements along the aerial axis. No longer is the earthed portion of the layout in the Flat Top configuration.

(d) The aerial portion of the aerial-ground structure serves as a loaded section of transmission line, providing energy exchange to the individual loading sections and related earthed elements. This aerial configuration remains as with Alexanderson systems. However, the undivided ground portion of Marconi/Alexanderson design now exists as an aerial counterpoise above the loaded aerial portion of the antenna-ground system. Hence, the Flat Top has become inverted, with the ground portion above the aerial portion of the system. The upper Flat Top configuration serves to neutralize the electric wave propagation in the space external to the mass of the earth. This neutralizing aerial confines the electric wave propagation of the antenna sub-system to the interior of the earth. The actual Alexanderson aerial is reduced to a loaded transmission line, unbalanced with respect to an elevated ground plane. No external electric wave propagation exists.

(e) A loaded transmission system is an analog system. Loading in its general form is a sequential series of alternate sections of real transmission line and of artificial transmission line. The artificial lines are analog equivalents of real propagation. Hereby the propagation on the aerial can be chosen at will through the interaction of real propagation with artificial (imaginary) propagation. The entire aerial becomes an analog network of real and imaginary parts, analogous to a complex wave propagation within the earth.

(f) Development of the telluric wave antenna centers upon the control of the phase relation or lead-lag time element, along the row of vertical earthed elements. The reflection at the surface of the earth of the standing and traveling telluric waves within produce specific images of phase displacement upon the surface of the earth. That is, the telluric waves develop specific points at the surface boundary. An analog of these waves is reproduced by the aerial portion of the antenna and connected with the earthed portion to facilitate the exchange of energy with the real wave through its projection upon the surface of the earth. The phase displacements of the individual earthed elements are now in mutual relation with the displacements of the telluric wave. The described antenna sub-system is attuned to the wave propagation within the interior of the earth.

(5) General Theory of Complex Electric Waves.

(a) Any electric wave is the product of the superposition of a magnetic field of induction and a dielectric field of induction. The pair of fields each represent the storage of electric energy within the structure of the field, magnetic or dielectric. Electric waves result from the exchange of electric energy between dielectric and magnetic fields of induction. The displacements of these inductions with respect of phase and distance determines the character of the resultant electric wave. Complex displacements give rise to complex electric waves.

(b) In common use are those electric waves that propagate along the axis of a system of two or more electric conductors. In this form of electric wave the magnetic and dielectric fields are both perpendicular to the axis of the system of electric conductors. The magnetic and dielectric fields are perpendicular to each other. Hence, the magnetic and dielectric fields of induction travel broadside or transverse to the propagation of the resultant electric wave along the electric conductors. The proportion of magnetic induction with respect to the portion of dielectric induction within an electric wave of this form is a numerical constant. This constant is numerically equivalent to the velocity of light in the space between the electric conductors. Also, it is a transverse electric wave, which propagates as a velocity, this velocity equivalent to the velocity of light in the space between the electric conductors. This common form of electric wave is called a transverse electro-magnetic wave (T.E.M.). The magnetic and dielectric fields are transverse to wave propagation.

(c) A complimentary electric wave exists in quadrature with the transverse electro-magnetic wave. Where the transverse wave propagates along the axis of the electric conductors, the quadrature wave propagates perpendicular to the axis of electric conductors. This conjugate electric wave is in space quadrature with the T.E.M. wave in any system of two or more electric conductors. As with the T.E.M. wave, this quadrature wave is the product of the superposition of the magnetic field and dielectric field of induction. With the quadrature electric wave the pair of fields of induction are co-liner or longitudinal to the direction of electric wave propagation. Hence, the magnetic field, dielectric field and electric wave propagation are all in space quadrature with the axis of the system of electric conductors. This quadrature electric wave is called the lonmgitudinal magnetic-dielectric wave (L.M.D.). The proportion of magnetic induction to the proportion of dielectric induction is <u>not</u> numerically equivalent to the velocity of light, nor is the dimension of propagation a velocity. That is, the longitudinal electric wave is not of the dimensions of unit length per unit time, as was the transverse electric wave. With the longitudinal electric wave the dimension of propagation is that of per unit length per unit time (per unit length-time). This propagation may be called a counter-velocity, representing propagation of electric waves through counter-space of per unit length.

(d) In the general case of telluric electric waves, the transverse wave propagates from a point of origin to distant locations through space of unit length over a period of unit time. The longitudinal wave propagates within the magnetic and dielectric fields themselves, within the point of origin, through a counterspace of per unit length over a period of per unit time. These two distinct forms of electric waves exist in a conjugate relation to each other. Hereby a complex electric wave propagates on a system of two or more electric conductors, with a real part, the T.E.M. wave and an image (imaginary) part, the L.M.D. wave. Their product is a complex quantity in the dimension of space. Thus the telluric electric wave is a complex electric wave consisting of a radiation component (T.E.M.) and a field of induction component (L.M.D.) in quadrature relation.

(6) Harmonic Structure of Transverse and Longitudinal Waveforms

(a) In the propagation of transverse electro-magnetic waves a progressive phase lag or delay results as the wave propagates outward from its origin, along the propagating structure. This results in an increasing phase shift or time lag for increasing frequency of energy exchange. For finite, resonant systems of electric conductors this phase lag is in unit integral multiples of quarter cycle delays. These delay factors result in harmonics of the cycle of energy exchange within the system of electric conductors. For example, Fo, 3Fo, 5Fo, etc. as this harmonic series progresses each harmonic becomes progressive diminished in amplitude. For example, Ao, 1/3 Ao, 1/5 Ao, etc.

(b) The harmonic series is contrary for the condition of longitudinal magneto-dielectric waves. In this case a progressive phase lead is produced as the wave propagates inward from its origin, within the propagating structure. This results in an increasing phase shift or time lag for decreasing frequency of energy exchange. For finite, resonate systems of electric conductors the phase shift is in unit differential divisions of quarter cycle advances. These advance factors result in the production of harmonics of the cycle of energy exchange. These harmonics exist as a series of divisions upon the fundamental frequency of energy exchange. For example, Fo, 1/3 Fo, 1/5 Fo, etc. As this harmonic series progresses the amplitude of each harmonic is progressively diminished as with the T.E.M. wave. For example, Ao, 1/3 Ao, 1/5 Ao, etc.

(c) The generalized, complex electric wave is the superposition of the time periods of T.E.M. propagation and its conjugate, the time periods of L.M.D. propagation. The resultant electric wave is a complex quantity in the domain of time, as well as the domain of space. Where in the space domain it is unit length for T.E.M. and per unit length for L.M.D., it is in the time domain unit time for the T.E.M. and per unit time for the L.M.D. with respect to harmonic production. The complex electric wave is the product of a progressive harmonic series and of a degressive harmonic series. Hereby the wave structure can be proportioned to produce a variety of electrical transient impulses with respect to time as well as space.