III General Theory of Telluric Electric Wave Transmission and Reception

• (1) Introduction

- (a) The reception and transmission of telluric electric waves employs methods and principles unlike those found in conventional electro-magnetic wave systems. Electric wave propagation within the interior of the earth is of a character much different than that propagation in the space exterior to the solid mass of the earth. The space within the mass of the earth is complicated by various degrees of magnetic permeability, dielectric permittivity, conductivity and resistivity, all these of various magnitudes for various directions. Further is the presence of significant static magnetic and static dielectric fields of induction.
- (b) With conventional electro-magnetic structures the principles of wave reception are 0 equivalent to the principles of wave transmission, this is known as the law of reciprocity. In part this law can be applied to the various sub-systems that form the telluric wave systems. However, certain antenna configurations depart from the reciprocity law in that the propagation factor for reception must lag in phase behind the electric wave to be received, where as the propagation factor for transmission must lead in phase ahead of the electric wave to be transmitted. This situation is analogous to the alternating current induction machine. For a given frequency in radians per sec. of electric excitation to the field of the induction machine the frequency in radians per second of rotation must fall behind the excitation frequency for the induction machine to receive energy as a motor. Conversely the frequency of rotation must push ahead of the excitation frequency for the induction machine to transmit energy as a generator. In this situation the angular frequency of excitation is of unit value or represents a relative condition of rest. The angular frequency of rotation has a relative negative or positive value, for positive or negative power flow respectively. For the antenna the situation is the same. The lagging velocity for receive and the leading velocity for transmit, this relative to the velocity of the electric wave in the medium of transmission or reception.
- (c) An important condition for the transmission and reception of telluric electric waves is a single wire or uni-polar connection to the solid mass of the earth. Electro-magnetic transmission and reception requires a multi-polar or multiple wire connection, two wire being common. It is required for telluric wave operation that the antenna sub-system be self referencing, that is the antenna sub-system not grounding in the usual sense, since ground is now an active terminal. There can be no second wire since there is nothing to connect it to. Hence, the need for a single wire or uni-polar antenna characteristic.
- (d) In the transmission and reception of telluric electric waves two departures exist with regard to the transmission and reception of electro-magnetic waves:

- 1) The law of reciprocity is not applicable to the transfer of energy between the telluric wave and the antenna sub-system.
- 2) The boundary condition or circuit law is not applicable to the connection of the antenna to the solid mass of the earth.
- (e) Such electrical conditions, once common in early wireless development, have become largely unknown. Two principal systems emerged from this era having the proper characteristics for telluric electric wave applications:
 - 1) The oscillation transformer as developed by Nikola Tesla, 1900
 - 2) The multiple loaded aerial as developed by Ernst Alexanderson, 1919

• (2) The Oscillation Transformer

- (a) The first development in the wireless transmission of electric waves was a telluric system based upon the application of an antenna sub-system known as the oscillation transformer. This transformer is a single winding coupled magnetically to an external resonant structure. Transformer operation resembles a constant current or ballast transformer. The single winding of the oscillation transformer resembles that of a simple reactance coil, however, only a single lead exists for communicating energy in and out of this coil structure. It is a single wire, uni-polar connection. The second lead of the coil is only connected to a small free space electro-static condenser.
- (b) In the operation of the oscillation transformer the winding is not a simple reactance coil and magnetic field of induction. The dielectric field of induction now plays an important role, as energy now resides in the dielectric field in addition to energy residing in the magnetic field. In oscillation transformer operation the total energy divides evenly between the magnetic field and dielectric field of induction. The superposition of these two fields of induction give rise to complex electric waves. The oscillation transformer winding, thus operates as a wave guide structure, giving rise to electric waves through the exchange of magnetic and dielectric energy.
- (c) Complex electric waves exist on the oscillation transformer winding during its resonant exchange of energy between the two fields. These standing waves produce a phase displacement in *time* cycle of energy exchange and in *length* along the winding structure. The displacements exist as a hysteresis cycle, displacing the cause-effect relationship. This results in the transformer winding operating as a uni-polar system.
- (d) The resonant structure coupled to the oscillation transformer winding is a simple magnetic reactance coil in a resonant relation with a simple electrostatic condenser. This circuit is proportioned to have minimal dissipative losses, that is it has a large magnification factor. This circuit provides the two wire connection for the supply or abstraction of energy to or from the oscillation transformer winding and its uni-polar connection to the earth.

- (e) In conjunction with the coupled resonant circuit the oscillation transformer winding serves as a phase transformer. This phase transformation provides the basis conversion from a multi-phase to a uni-phase connection. This phase transformation provides the basis conversion from a multi-phase to a uni-phase connection. This provides the single wire connection for the telluric electrical waves, transforming this to the multiple wire connection to a network sub-system.
- (f) The complex electric wave produced by the resonant electric fields of the oscillation transformer winding is analogous to those electric waves, which exist within the interior of the earth. This complex electric wave in the winding is the resultant of the superposition of transverse electric waves of a specific velocity and of longitudinal waves of a specific counter-velocity. This pair of electric waves propagate within the electric field of the winding. This winding can be proportioned to be attuned to the complex electric wave propagation within the interior of the earth. The winding becomes an analog of the specific telluric waves to be transmitted or received.
- (g) The transient impulses produced by the oscillation transformer are of analogous form to the transient impulses resulting from telluric wave propagation within the earth. In this manner the oscillation transformer responds as does the network sub-system previously described. The high and low pass functions are a direct result of transformer actions. Hereby the oscillation transformer serves as the network sub-system in addition to serving as the antenna sub-system. Thus the oscillation transformer in itself serves as a system for the transmission and reception of telluric electric waves.
- (h) The principal drawback in the application of the oscillation transformer to telluric waves is the inability to respond to a wide range of signal frequencies. Also is the lack of directivity in the spatial distribution of its response. Thus in the application of the oscillation transformer to telluric waves it can be proportioned to respond only to telluric waves of a single frequency and its related harmonic structure, that is to one specific transient electric wave form. Transformer response is to telluric waves from all directions, it having no directional character. This limits the use of the oscillation transformer to specific communication or broadcast functions and thus prohibits its use for broadband or generalized transmission or reception functions.

• (3) The Multiple Loaded Aerial

(a) Following the development of the oscillation transformer was its application to the system of wireless trammission developed by Guglielmo Marconi (1910). During its development by Nikola Tesla the wave guide and uni-polar properties of the oscillation transformer were not fully understood. Tesla repeatedly attempted to force the winding to operate as a simple magnetic reactance coil. The importance of the dielectric field of the winding and its complex relation to the magnetic field were to be missed by Tesla and his contemporaries. This situation would be further compounded by the efforts of Marconi.

- (b) The application of the oscillation transformer to the transmission and reception of telluric waves was under patent protection by Nikola Tesla. For Marconi to proceed with his wireless development, significant alterations had to be made. In the telluric wave system of Nikola Tesla the oscillation transformer alone served as the basic system for the transmission or reception of electric waves. Marconi would make important changes to the Tesla system in order to secure a wireless patent of his own. The basic modification was the extreme enlargement of the electro-static capacity of the free terminal of the oscillation transformer winding. An aerial-ground structure known as the Marconi "Flat Top", of considerable extent, was connected as a basic condenser to the oscillation transformer winding. The electro-static capacity of this aerial-ground structure greatly exceeded that of the oscillation transformer winding. Hereby the function of the oscillation transformer was reduced to that of a basic magnetic reactance coil. Now unable to resonate with the winding dielectric field, the winding lost the ability to operate as a phase transformer. It now operates as a di-polar or two wire system.
- o (c) The aerial portion of the Marconi Flat Top was positioned over a similar structure in the ground. The length of this aerial-ground system several times larger than the width, this forming a large strip-line transmission structure. Thus the aerial-ground structure is a electrically short section of electro-magnetic transmission line. Within the electro-magnetic field of this section of line, a very large reactive power flow exists, this in an oscillitory energy exchange with the transformer. The coil and aerial-ground structure reduce to a basic resonant circuit. The power flow in this circuit contributes little to the ability of the Flat Top to transmit or receive electric waves. The Flat Top derives its ability to transmit or receive principally from it external dielectric field of induction. The lag of phase along the length of the Flat Top produces a small portion of external electro-magnetic activity and resulting waves. Hence, the Marconi Flat Top aerial-ground system is an ineffective structure for both electromagnetic and telluric electric waves.
- (d) The large reactive power flow within the confined portion of the Flat Top represents a useless or parasitic power flow. Its loading upon the oscillation transformer renders the winding a reactance coil. Therefore, the confined electro-magnetic field of induction inhibits the operation of this aerial-ground system in the transmission or reception of telluric electric waves.
- (e) While Marconi resorted to simple terminal impedance methods to minimize the effects of this reactive power flow, the basic situation remained unchanged. Ernst Alexanderson, while employed by the General Electric Company and the U.S. Navy (1919), developed a significant advancement in the Flat Top system. This development became the Alexanderson multiple loaded aerial. This aerial-ground system finds important applications to the transmission and reception of telluric electric waves.
- (f) The Alexanderson system is a direct adaption of the Marconi Flat Top. The basic external geometry is unchanged. However, the aerial and the ground elements of the strip-line

configuration are sectionalized into a series of sub-section elements. Loading elements are inserted in the transitions between sub-section elements. The Alexanderson principle utilized this sequential loading to cancel or neutralize the reactive electro-magnetic power flow of the Flat Top system. The result is the aerial-ground system becomes a non electro-magnetic structure, with the dimensions of velocity and wavelength becoming undefined.

- (g) The Alexanderson system is no longer the simple strip-line of Marconi, but has become a complex system of alternate, sequential sections of transmission and loading structures. This configuration is analogous to a loaded long distance telephone line. The Alexanderson system has rendered the strip-line of Marconi a *wave-guide* type structure. The superposition of the magnetic field and the dielectric field in this wave-guide give rise to complex electric waves as with the oscillation transformer of Tesla. Hereby the Alexanderson system enables telluric wave transmission and reception.
- (h) The basic oscillation transformer winding exists in multiple with the Alexanderson system, each being connected at each of the sequential loading sections. These windings now operate unhampered by reactive power flow. Operation of these phase transformer windings in multiple allows for directional operation, unlike a single unit. In addition, in conjunction with the loading elements, the windings in multiple allows for a band pass characteristic to be established. Hereby the Alexanderson multiple loaded aerial-ground system overcomes the principal limitations of the oscillation transformer system of Tesla, the lack of directivity and bandwidth.

• (4) Development of the Alexanderson System for the Propagation of Telluric Waves.

- (a) The basic Alexanderson system can be developed further for adaption to the propagation of telluric waves. Alexanderson would follow the path that Marconi followed from the Tesla system. The Alexanderson system was operated as a di-polar configuration for the propagation of electro-magnetic waves. This led to the extinction of the Alexanderson system as well as the systems from which it developed. Advancing the Alexanderson concept one step ahead, while retaining the original uni-polar concept of Tesla, results in an aerial-ground sub-system of perfect adaptability to the transmission and reception of telluric waves.
- (b) In the layout of the Alexanderson system upon that of Marconi, the earthed portion of this system basically remained unchanged. This portion continued to operate as a single grounded conductor under the aerial portion of the system. No significant phase or potential differences exist along the continuous earthed portion of the Alexanderson system. Therefore, no electric waves can exist along this length of grounded conductor. This earthed portion of the aerial-ground system acts as a single ground electrode and can propagate telluric waves only in the manner of a single point source, as with the Tesla system. During the period of history during which the Alexanderson system existed, it was considered as a system for the propagation of electro-magnetic waves. This related to the

vertical conductors rising from the grounded loading sections to the aerial structure above. The electric current related to this conductor gives rise to the propagation of electromagnetic waves. However, the energy of these waves exists as a small portion of the total electric wave propagation of the Alexanderson aerial-ground system. Alexanderson as well as Marconi engineers understood that the Flat Top aerial and its adaptation by Alexanderson operated as an antenna for the propagation of *electro-static*, rather than electro-magnetic waves. Therefore, the Alexanderson aerial operates as a system for the transmission and reception of dielectric waves through its external dielectric field. Part of this dielectric field of induction is directed by the earthed ground structure into the interior of the earth. This induction gives rise to the propagation of telluric waves in a manner similar to that of Tesla.

- c) (c) The Alexanderson aerial-ground system is an advancement upon the Marconi system. The Marconi system is an adaptation of the Tesla oscillation transformer system. The antenna sub-system of the basic system for the propagation of telluric waves represents an advancement upon the system of Tesla, this lacking directivity and bandwidth and represents an advancement upon the system of Marconi/Alexanderson, which primarily propagated waves exterior to the mass of the earth. The advancement upon the Alexanderson system is the *elimination of the external propagation* of electric waves. Advancement centers upon the *earthed* portion of the aerial-ground system. Unlike the Alexanderson configuration loading is *divided in a balanced fashion* between both the aerial and earthed portions of the system. Hereby complex electric waves within the interior of the earth can be developed. The elements of the earthed portion of this system operate each independent of the other with no inter-connection. Each element consists of a vertical section projecting within the mass of the earth. The earthed portion of the aerial-ground structure exists as a sequential row of vertical earthed elements along the aerial axis. No longer is the earthed portion of the layout in the Flat Top configuration.
- (d) The aerial portion of the aerial-ground structure serves as a loaded section of transmission line, providing energy exchange to the individual loading sections and related earthed elements. This aerial configuration remains as with Alexanderson systems. However, the undivided ground portion of Marconi/Alexanderson design now exists as an aerial counterpoise above the loaded aerial portion of the antenna-ground system. Hence, the Flat Top has become inverted, with the ground portion above the aerial portion of the system. The upper Flat Top configuration serves to neutralize the electric wave propagation in the space external to the mass of the earth. This neutralizing aerial confines the electric wave propagation of the antenna sub-system to the interior of the earth. The actual Alexanderson aerial is reduced to a loaded transmission line, unbalanced with respect to an elevated ground plane. No external electric wave propagation exists.
- (e) A loaded transmission system is an analog system. Loading in its general form is a sequential series of alternate sections of real transmission line and of artificial transmission line. The artificial lines are analog equivalents of real propagation. Hereby the propagation

on the aerial can be chosen at will through the interaction of real propagation with artificial (imaginary) propagation. The entire aerial becomes an analog network of real and imaginary parts, analogous to a complex wave propagation within the earth.

(f) Development of the telluric wave antenna centers upon the control of the phase relation or lead-lag time element, along the row of vertical earthed elements. The reflection at the surface of the earth of standing and traveling telluric waves within produce specific images of phase displacement upon the surface of the earth. That is, the telluric waves develop specific points at the surface boundary. An analog of these waves is reproduced by the aerial portion of the antenna and connected with the earthed portion to facilitate the exchange of energy with the real wave through its projection upon the surface of the earth. The phase displacements of the individual earthed elements are now in mutual relation with the displacements of the telluric wave. The described antenna sub-system is attuned to the wave propagation within the interior of the earth.

• (5) General Theory of Complex Electric Waves

- (a) Any electric wave is the product of the superposition of a magnetic field of induction and a dielectric field of induction. The pair of fields each represent the storage of electric energy within the structure of the field, magnetic or dielectric. Electric waves result from the exchange of electric energy between dielectric and magnetic fields of induction. The displacements of these inductions with respect of phase and distance determines the character of the resultant electric wave. Complex displacements give rise to complex electric waves.
- (b) In common use are those electric waves that propagate along the axis of a system of two or more electric conductors. In this form of electric wave the magnetic and dielectric fields are both perpendicular to the axis of the system of electric conductors. The magnetic and dielectric fields are perpendicular to each other. Hence, the magnetic and dielectric fields of induction travel broadside or transverse to the propagation of the resultant electric wave along the electric conductors. The proportion of magnetic induction with respect to the portion of dielectric induction within an electric wave of this form is a numerical constant. This constant is numerically equivalent to the velocity of light in the space between the electric conductors. Also, it is a transverse electric wave, which propagates as a velocity, this velocity equivalent to the velocity of light in the space between the electric conductors. This common form of electric wave is called a transverse electro-magnetic wave (T.E.M.). The magnetic and dielectric fields are transverse to wave propagation.
- (c) A complimentary electric wave exists in quadrature with the transverse electro-magnetic wave. Where the transverse wave propagates along the axis of the electric conductors, the quadrature wave propagates perpendicular to the axis of electric conductors. This conjugate electric wave is in space quadrature with the T.E.M. wave in any system of two or more electric conductors. As with the T.E.M. wave, this quadrature wave is the product of the

superposition of the magnetic and dielectric fields of induction. With the quadrature electric wave the pair of fields of induction are co-liner or longitudinal to the direction of electric wave propagation. Hence, the magnetic field, dielectric field and electric wave propagation are all in space quadrature with the axis of the system of electric conductors. This quadrature electric wave is called the longitudinal magnetic-dielectric wave (L.M.D.). The proportion of magnetic induction to the proportion of dielectric induction is not numerically equivalent to the velocity of light, nor is the dimension of propagation a velocity. That is, the longitudinal electric wave. With the longitudinal electric wave the dimension of propagation is that of per unit length per unit time (per unit length-time). This propagation may be called a counter-velocity, representing propagation of electric waves through counter-space of per unit length.

(d) In the general case of telluric electric waves, the transverse wave propagates from a point of origin to distant location through space of unit length over a period of unit time. The longitudinal wave propagates within the magnetic and dielectric fields themselves, within the point of origin, through a counterspace of per unit length over a period of per unit time. These two distinct forms of electric waves exist in a conjugate relation to each other. Hereby a complex electric wave propagates on a system of two or more electric conductors, with a real part, the T.E.M. wave and an image (imaginary) part, the L.M.D. wave. Their product is a complex quantity in the dimension of space. Thus the telluric electric wave is a complex electric wave consisting of a radiation component (T .E.M.) and a field of induction component (L.M.D.) in quadrature relation.

• (6) Harmonic Structure of Transverse and Longitudinal Waveforms

- (a) In the propagation of transverse electro-magnetic waves a progressive phase lag or delay results as the wave propagates outward from its origin, along the propagating structure. This results in an increasing phase shift or time lag for increasing frequency of energy exchange. For finite, resonant systems of electric conductors this phase lag is in unit integral multiples of quarter cycle delays. These delay factors result in harmonics of the cycle of energy exchange within the system of electric conductors. For example, Fo, 3Fo, 5Fo, etc. as this harmonic series progresses each harmonic becomes progressive diminished in amplitude. For example, Ao, I/3 Ao, I/5 Ao, etc.
- (b) The harmonic series is contrary for the condition of longitudinal magneto-dielectric waves. In this case a progressive phase lead is produced as the wave propagates inward from its origin, within the propagating structure. This results in an increasing phase shift or time lag for decreasing frequency of energy exchange. For finite, resonate systems of electric conductors the phase shift is in unit differential divisions of quarter cycle advances. These advance factors result in the production of harmonics of the cycle of energy exchange. These harmonics exist as a series of divisions upon the fundamental frequency of energy exchange. For example, Fo, 1/3 Fo, 1/5 Fo, etc. As this harmonic series progresses

the amplitude of each harmonic is progressively diminished as with the T.E.M. wave. For example, Ao, I/3 Ao, I/5 Ao, etc.

(c) The generalized, complex electric wave is the superposition of the time periods of T.E.M. propagation and its conjugate, the time periods of L.M.D. propagation. The resultant electric wave is a complex quantity in the domain of time as well as the domain of space. Where in the space domain it is unit length for T.E.M. and per unit length for L.M.D., it is in the time domain unit time for the T.E.M. and per unit time for the L.M.D. with respect to harmonic production. The complex electric wave is the product of a progressive harmonic series and of a degressive harmonic series. Hereby the wave structure can be proportioned to produce a variety of electrical transient impulses with respect to time as well as space.



Archetypical Systems

÷ 1 1 + SPACE FIG D 40 TESLA SYSTEM (+)SPACE CURRENT Ls Cs LEADS GROUND SECONDARY CIRCUT CURRENT BY 90° M Ð al= Cp ESUPPLY PRIMARY C Lp CARCUIT at Le EARTH SPACE il mil Is FIGE MARCONI SYSTEM (+) SPACE CURRENT Cs SECONDARY & LS EQUAL TO GROUND CURRENT CUECHT $I_e = -I_s$ PRIMARY a=1 ESPER Cp EURCUIT ELP : Ie SYMBOLIC COMPARISON OF ELECTRO-STATIC 1 WIRELESS SYSTEMS EARTH 171 TESLA VS MARCONT L. COIL 1 1 CONDENSER C. (BREAK) CHOPPER

9-9 Symbolic Comparison of Electro-Static Wireless Systems - Tesla vs. Marconi



Electro-Static Condenser



Antenna Diagram

- 1)
- A) The electrostatic capacity of the Bolinas array can be divided into two distinct categories:
 - That part of the electrostatic field confined between the elevated capacity and the ground plane:

$$C_{z} = 1.5 \times 10^{-8}$$
 farad 0.015 uFd

 That part of the electrostatic field which extends from the elevated capacity to space:

$$C_s = 3.5 \times 10^{-9}$$
 farad 3500 pFd.

And therefore a total electrostatic capacity of:

$$C_o = 1.9 \times 10^{-8}$$
 farac

With a ratio of:

$$C_{s}: C_{c} = 0.35$$

And a transmission efficiency of:

$$C_s: C_o = 19$$
 percent.

B) The electrostatic potential is given as:

$$E_0 = 100$$
 kilovolts.

With an angular velocity of:

Energy is supplied to this potential at a rate of:

P = 200 kilowatts.

- C) For a peak potential of 100 kilovolts, the two electrostatic fields are:
 - 1) The confined field:

 $\Psi = 1.8 \times 10^{10}$ lines of force

W = 32 watt-second

2) And the transmitted field:

 $\gamma = 4.2 \times 10^9$ lines of force W = 7 watt-second

D) For potential variation of 1.2 x 10 radians per second, the power flow of the two electrostatic fields are:

1) Confined power flow:

$P_c = 17 \times 10^{\circ}$	volt-amperes	
$X_{c} = 6 \times 10^{2}$	sec. per farad (ohm)	

2) And the transmitted power flow:

 $P_s = 4.8 \times 10^6$ volt-amperes $X_s = 2 \times 10^3$ sec. per farad (ohm)

E) The total electric current transmitted into the earth is hence given:

 $l_o = 48$ amperes

With a transmission loss of 200 kilowatts and a corresponding electromotive force of:

 $E_o = 4200$ volts

F) For the entire array the total power flow is:

Scale 2

 $P = 21 \times 10^6$ volt-amperes

And for a dissipation rate of 200 kilowatts, the power multipaction factor is thus given:

 $\varphi = 100 x$ dimensionless

II) The entire array is divided into three distinct section elements: element 1 and element 2, and a third terminal element

A) The mid-section elements are of the following electrical dimensions:

1) Electrostatic capacity to space:

$C_s = 1.4 \times 10^{-9}$	farad	1400 pFd
$P_{s} = 1.7 \times 10^{6}$	volt-amperes	
$I_{s} = 17$	amperes	
$X_{s} = 6 \times 10^{\circ}$	sec. per farad	6Ka

2) Electrostatic capacity to ground:

$C_e = 4.9 \times 10^{-9}$	farad	4900 pFd
$P_c = 5.6 \times 10^{\circ}$	volt-amperes	
$l_{e} = 56$	amperes	
$X_c = 1.8 \times 10^3$	sec. per farad	1.8Ka

3) Electromagnetic inductance:

 $L = 1.3 \times 10^{5}$ Henry $X_{L} = 16$ Henry/sec. (OHM)

 The electro-motive force developed by the electromagnetic induction of the element half-section L/2 is given by the relation:

 $E_{1}/I_{2} = X$ E = 550 volts

And therefore, the power flow of this induction:

 $E_{x}I_{T} = P = 40 \times 10^{3}$ volt-amperes

And thus the ratio of magnetic to electrostatic power flow is:

 $P_{L}: P_{O} = 40:7300 = 1.5 \text{ percent}$

Scale 3

B) Having derived the electromagnetic and the electrostatic coefficients of the elemental sections, the electromagnetic propagation coefficients are thus:

 Z_{c} = transmission impedance of confined electromagnetic wave Z_{c} = 173 ohm Z_{s} = transmission impedance of un-confined electromagnetic wave Z_{s} = 316 ohm

And likewise:

 V_o = transmission velocity of confined propagation

 $V_0 = 3 \times 10^{10}$ cm./sec.

 V_s = transmission velocity of un-confined propagation

 $V_s = 5.6 \times 10^{10}$ cm./sec.





These coefficients are defined by the established conventional physical dimensions



Let the shunt coefficients be combined by the relation

$$M + u^2 C = M + \frac{1}{(k\omega)^2} C \tag{5}$$

And let the series coefficients be combined by the relation

$$K - u^2 L = K - \frac{1}{\left(k\omega\right)^2} \tag{6}$$

Where the factor

$$u^{-2} = (k\omega)^{+2} \qquad \text{per second}^2 \tag{7}$$

and

$$\omega = 2\pi F$$
 per second

 $k^4 = -1$ A dimensionless unit























